Charge Dependent Effects On Nuclear Forces

by

Al-Aithan Thamer Abdul-Muhsen

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
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Dedicated to my family

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خلاصـــــة

ان دراسة عدم الاعتماد على الشحنة وتناظر الشحنة عن طريق بعد التشتت عندما تكون قوة الدفع الزاوي لتفاعل وحدات القوة النووية (النيترون والبروتون) صفرا فانها تعطى دلالة على تأثير الشحنة على القوة النووية . لقد تم فى هذا البحث اعتبار جميع المساهمات المحتملة المعروفة لتفسير الفروقات التجريبية . من جهة أخرى تم تقسيم هذه المشكلة الى جزئين ، الأول هو التأثير الكهرومغناطيسى المباشر والثاني التأثير الكهرومغناطيسى غير المباشر . تم التركيز في هذا البحث على التأثير في حالة تناظر الشحنة والناتج عن فرق الكتلة بين الكوارك العالي والكوارك الواطي والذي يسبب اختلاط جزئ بايون مع جزئيي ايتا عن طريق الكم الكروموداينامیکی . ولقد حسب هذا التأثير من خلال حساب تأثير الكم الكروموداينامیکی على ثابت الدمج لوحدات القوة النووية عند تبادل جزئ البايون ومن ثم ماينتج من تأثير على بعد التشتت .

لقد تم التوصل الى أهمية مساهمة تبادل جزيئين في القوة النووية وكذلك تحديد ثابت دمج الوحدة النووية مع جزئ ايتا لتكون صغيرة جدا ومحددة بالقيمة التجريبية لتناظر الشحنة .

ABSTRACT

The study of charge independence and charge symmetry on the $^1S_0$ N-N scattering lengths provides a test of charge dependent effects of nuclear forces. All possible known contributions to the scattering lengths are considered to account for the experimental discrepancies. This problem was broken into two parts, the first considering the direct electromagnetic and the second the indirect electromagnetic effects. The effect on the charge symmetry breaking correction due to the isospin breaking part of the QCD Hamiltonian, determined by the up-down current quark mass difference, on the difference of scattering lengths for charge symmetry is determined. This is done by first calculating the effect of QCD on the pion-nucleon coupling constants via isospin mixing, giving rise to CSB one pion exchange (OPEP) and two pion exchange (TPEP) potentials. The importance of CSB-TPEP is emphasized. The eta-nucleon coupling constant appears to be constrained by experimental values of CS to be very small.

CHAPTER ONE

INTRODUCTION

The strong nuclear force is one of the three forces in nature, the other two being the gravitational and electroweak forces. For a long time it has been realized that the strong nuclear force, as represented by the nucleon-nucleon (N-N) interaction, is charge independent. But its charge independence is broken by certain small effects which violate the isospin symmetry of this interaction. In this thesis, the problem of the charge dependence of the nuclear force is studied in some detail.

The problem of the charge independence (CI) and charge symmetry (CS) of nuclear forces has a distinguished history in modern physics. CI and CS can be expressed as isospin invariance of the strong nuclear force. Isospin symmetry is one of the first internal symmetries which are independent of space-time symmetries. This problem, at the same time, provides an interplay between nuclear physics and particle physics

(ref.9). The explanation of discrepancies between observed parameters "which are indicators of the strength of the nuclear force" and those predicted by the CI and CS of nuclear forces in terms of effects which break the isospin symmetry of nuclear forces continues to be studied in the literature. In the past it was attempted to solve the problems by considering QED, but later QCD was also considered to explain the small observed charge dependence of nuclear forces. The isospin symmetry of the strong interaction between nucleons may be broken by electromagnetic forces either directly or indirectly or by effects which do not appear to be of electromagnetic origin but nevertheless break isospin symmetry, e.g. the up-down quark mass difference in QCD. The breaking of CI and CS are useful guides in understanding the symmetry breaking of the nuclear, or strong, interaction.

As is well known the $^1S_0$ pp, nn and pn scattering lengths provide a test of the charge symmetry (CS) and charge independence (CI) of nuclear forces. CS, implying vanishing of the difference $\delta a_{CSB} = (a_{pp} - a_{nn})$, holds within the experimental error ($\delta a_{CSB} \approx 0.55 \pm 0.46$ fm). CI, which would imply

the vanishing of $\Delta a = [\ \{\frac{a_{pp} + a_{nn}}{2}\} - a_{np}\ ]$, is broken by the measured[1] $\Delta a = (\ 5.5 \pm 0.3\ fm\ )$.

The understanding of δa and Δa in terms of charge dependent effects has a long history (ref.2). The study of charge independence breaking (CIB) and charge symmetry breaking (CSB) effects have been reconsidered (ref.3) more carefully with the progress and development of particle physics. CIB, as exemplified by the above value of Δa, is almost solved by taking into account the $\pi^{\pm} - \pi^{0}$ mass difference and electromagnetic (e.m.) corrections to the pion-nucleon coupling constants in the one pion exchange potential (OPEP) and the two pion exchange potential (TPEP), and by the photon-pion exchange potential. The other possible contributions to CIB, such as short range effects due to six quark contributions, exchange of heavy I = 1 mesons etc., have been found to be very small.

The situation with respect to CSB is not so clear. Apart from many small effects such as $\rho^{0} - \omega^{0}$ particle mixing, $\pi^{0} - \eta, \eta'$ particle mixings, and $\gamma\pi$ boson exchange, an important contribution to CSB arises from electromagnetic corrections to the $\pi^{0}pp$ and $\pi^{0}nn$ coupling constants.

In this thesis, we will concentrate on charge symmetry breaking (CSB) due to QCD, its effect on the coupling constants, and also on the scattering lengths. These effects are due to the up-down current (not constituent) quark mass difference, $H_{QCD} = \frac{1}{2}(m_u - m_d)(\bar{u}u-\bar{d}d)$. Many authors (refs.2-4) found that the one boson exchange potential, OBEP, due to QCD gives a very small contribution to the scattering length. On the other hand, the two-pion exchange potential TPEP-QCD contributions were not considered and some argued that they are small compared to OBEP (ref.5). However, in our work we will show the importance of TPEP-QCD.

The approach we wish to follow is to estimate the effect of Δm_q on the pion-nucleon coupling constants, where Δm_q appears as the current quark mass difference in the basic Lagrangian or Hamiltonian:

$$H = H_{QED} + H_{\Delta m_q} . \qquad (1.1)$$

Here H_{QED} is the usual electromagnetic current-current Hamiltonian due to an internal photon loop while

$$H_{\Delta m_q} = -\frac{1}{2}\Delta m_q (\bar{u}u-\bar{d}d) . \qquad (1.2)$$

The effect of H_{QED} on the pion-nucleon coupling constants has previously been calculated by various authors (see ref.2 for a review). What about the effect of Δm_q on the pion-nucleon coupling constants ? One way such an effect can be taken into account is by observing that $H_{\Delta m_q}$ can give rise to $\eta - \pi^0$ mixing. The effects due to such a mixing on the one meson exchange potential and that of the latter on the scattering lengths have been calculated in ref.7. However, if one looks on the third and fourth lines of table 4.1, one observes the important fact that there is an almost complete cancellation between Δa_{cc}^{OPEP} and Δa_{cc}^{TPEP} , i.e. to say when one considers only the effect of H_{QED} on the π-N coupling constants there is an almost complete cancellation from contributions arising from the one pion exchange and two pion exchange potentials. On the other hand, the calculation of ref.7 in effect takes into account modifications due to $\eta - \pi^0$ mixing in OPEP only. In view of the above remarks it is important that the effect of Δm_q on the $\eta - \pi^0$ mixing should also be studied in TPEP. Thus it should be more consistant to add the effect of (1.2) on the

π-N coupling constants that of H_{QED} (which has previously been calculated) and then study the combined effect of (1.1) on Δa_{cc}^{QPEP} and Δa_{cc}^{TPEP} in order to see whether the cancellation between the two still persists.

For a complete study of this problem, we will firstly review the theory. In chapter two, the definition of isospin, scattering length theory, and effective range theory are considered in some detail. In chapter three we will review the experimental observations of scattering lengths and effective ranges for $^1S_0$ pp , nn and np scattering. In chapter four we will consider the charge dependent effects which has been previously calculated. In chapter five we will consider computation of various effects. Finally we will consider corrections due to QCD and then present our conclusions and discuss our results.

CHAPTER TWO

THEORETICAL REVIEW

In this chapter the theory and definitions which are needed to explain CI (charge independence) and CS (charge symmetry) and their breaking are explained in some detail. The relation between the scattering length and the nucleon-nucleon potential is also derived. Contributions from different charge dependent effects leading to CIB and CSB will also be considered.

2.1-ISOSPIN

Isospin is introduced to incorporate the CI of nuclear forces. It is considered to be a quantum number in an internal space called Isospin space. The neutron state $|n\rangle$ and proton state $|p\rangle$ are regarded to be two different states of the nucleon, so that

$$X_p = X_+ = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |p\rangle \qquad (2.1)$$

$$X_n = X_- = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |n\rangle \qquad (2.2)$$

define the fundamental (doublet) representation of the $SU_I(2)$ group generated by the isospin operator $\underline{I} = (1/2)\underline{\tau}$ where $I=(I_1, I_2, I_3)$ is a vector in isospin space and τ is the usual Pauli isospin matrix with components

$$\tau_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \tau_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{2.3}$$

These matrices obey the following commutation relations

$$[\tau_i , \tau_j] = \tau_i\tau_j - \tau_j\tau_i = 2i\varepsilon_{ijk}\tau_k, \tag{2.4}$$

Where

$$\varepsilon_{ijk} = \begin{cases} +1 & \text{even permutation of indices} \\ -1 & \text{odd permutation of indices} \\ 0 & \text{if any two of indices are equal .} \end{cases} \tag{2.5}$$

In other words I behaves similarly to the spin angular momentum so that we can use the eigenvalues of I^2 and I_3, where

$$I^2 = I_1^2 + I_2^2 + I_3^2 , \tag{2.6}$$

to label the states. Thus for the state $X_I^{I_3}$, called the isospin wave function,

$$I^2 X_I^{I_3} = I(I+1) X_I^{I_3} \tag{2.7}$$

$$\underline{I}_3 \, X_I^{I_3} = I_3 \, X_I^{I_3} \qquad (2.8)$$

I and I_3 are the eigenvalues of the isospin magnitude of I^2 and 3-rd component of I respectively. The eigenvalues of I^2 and I_3 for a nucleon are

$$I = \frac{1}{2} \quad , \quad I_3 = \pm\frac{1}{2}$$

i.e.

$$\underline{I}^2 \, X_N = \frac{1}{2}(\frac{1}{2}+1)X_N \, , \qquad N = n,p \qquad (2.9)$$

$$\underline{I}_3 \, X_n = -(\frac{1}{2}) \, X_n \qquad (2.10)$$

$$\underline{I}_3 \, X_p = (\frac{1}{2}) \, X_p \qquad (2.11)$$

Note that the choice of sign of the eigenvalues of the 3-rd component of I may be used reversely i.e.

$$\underline{I}_3 \, X_n = (\frac{1}{2})X_n$$

$$\underline{I}_3 \, X_p = -(\frac{1}{2})X_p \qquad (2.12)$$

From Eqs(2.10), and (2.11) by definition, I_3 distingushes between the neutron and proton. The electric charge operator Q is related to I_3 by,

$$Q = e\ (1/2 + I_3), \tag{2.13}$$

where e is the charge of the proton. This implies

$$Q\ |p\rangle = e(1/2 + 1/2)\ |p\rangle = e|p\rangle \tag{2.14}$$
$$Q\ |n\rangle = e\ (1/2 - 1/2)\ |n\rangle = 0$$

which gives the correct charge for each nucleon type.

The state of a system of nucleons is labelled by I and I_3, as well as other quantum numbers, i.e. it is expressed as

$$|\alpha, I, I_3\rangle \tag{2.15}$$

so that the neutron and proton states are represented by

$$|p\rangle = |\alpha, 1/2,\ 1/2\rangle \tag{2.16}$$
$$|n\rangle = |\alpha, 1/2, -1/2\rangle,$$

where α stands for all the other quantum numbers which are neccesary to distingush one state from others. Similar to the angular momentum vector, I_3 can take (2I+1) values given by:-

-I, -I+1,....., I-1, I.

Out of the components I_i (i=1,2,3) we can also form

raising and lowering operators:

$$I_{\pm} = I_1 \pm iI_2, \tag{2.17}$$

which for the case of the nucleon take the form

$$I_+ = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$I_- = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} . \tag{2.18}$$

The effect of these operators on the nucleon states, defined by Eq (2.12), is given by

$$\begin{aligned} I_+ X_p &= X_n \\ I_- X_p &= 0 \\ I_+ X_n &= 0 \\ I_- X_n &= X_p \end{aligned} \tag{2.19}$$

These operators change the neutron to proton or vice versa.

2.2-ISOSPIN INVARIANCE AND ITS BREAKING

Charge independence in a hadronic interaction means that the force between nucleons is independent of their electric charge. In other words the hadronic interaction is independent of the direction in isospin space. This leads to the fact that the hadronic interaction is invariant under rotations in isospin space, in complete analogy with ordinary angular momentum.

The unitary transformation corresponding to a rotation in isospin space can be expressed by the following unitary operator:

$$U_I(\alpha) = \exp(iI.\alpha), \qquad (2.20)$$

where α is the angle of rotation, I is the isospin operator. Isospin invariance means;

$$[H_h, U_I(\alpha)] = 0, \qquad (2.21)$$

where H_h is the hadronic interaction Hamiltonian, responsible for example for the N-N interaction. For an infinitesimal rotation

$$U_I(\alpha) = 1 + i\ I\ .\ \alpha \qquad (2.22)$$

$$[H_h, I] = 0 \ . \qquad (2.23)$$

Thus I is a conserved quantum number in hadronic interactions, and so is I_3, i.e.

$$\Delta I^2 = I_f^2 - I_i^2 = 0 \qquad (2.24)$$

$$\Delta I_3 = I_{3f} - I_{3i} = 0,$$

where f stands for the final state and i for the initial state.

Including the electromagnetic interaction we can write the complete Hamiltonian in the form

$$H = H_h + H_{em} \qquad (2.25)$$

where H_h is invariant under an isospin rotation, but H_{em} breaks the isospin invariance i.e.

$$[\ H\ ,\ I\ ] = [\ H_{em}\ ,\ I\ ] \neq 0, \qquad (2.26)$$

since as is clear from eqn.(2.3), H_{em} involving electric charge has selected a particular direction in isospin space. For example, the Coulomb potential between two nucleons which can be written as:

$$V_c^{ij} = \frac{e^2}{r} \left(\frac{1}{2}+I_3^i\right) \left(\frac{1}{2}+I_3^j\right) \qquad (2.27)$$

breaks isospin symmetry, i.e.

$$[\ V_c^{ij}\ ,\ I\ ] \neq 0, \qquad (2.28)$$

for the p-p interaction, but note that I_3 is still conserved. Another example is the mass Hamiltonian, H_m, if we take $m_n \neq m_p$. The Kinetic energy term is

$$\text{K.E.} = - \sum_{i=1}^{A} \nabla_i^2 \left[\frac{B_+^{(i)}}{2m_n} + \frac{B_-^{(i)}}{2m_p} \right] \qquad (2.29)$$

where K.E. stands for kinetic energy, A is the number of nucleons, m_p is the mass of the proton, m_n is the mass of the neutron, and $B_\pm = \frac{1}{2}(1 \mp I_3)$ are projection operators with the properties:-

B_+ projects out neutron

B_- projects out proton

Then by rearranging eqn.(2.29) we get

$$\text{K.E.} = - \sum_{i=1}^{A} \frac{\nabla_i^2}{2} \left[\frac{\left(\frac{1}{2} - I_3^i\right)}{2m_p} + \frac{\left(\frac{1}{2} + I_3^i\right)}{2m_n}\right] \qquad (2.30)$$

and thus

$$\text{K.E.} = -\sum_{i=1}^{A} \frac{\nabla_i^2}{2M}\left[1 - \frac{\Delta M}{M} I_3^i\right] + O\left(\frac{\Delta M}{M}\right)^2, \quad (2.31)$$

where $M = 1/2\ (m_n + m_p)$ and $\Delta M = (m_n - m_p)$. Thus it is clear from eqn.(2.31) that K.E. breaks isospin symmetry since $[\text{K.E.}, I] \neq 0$, because I_3 does not commute with I_1 or I_2. If, however ΔM is neglected, i.e. $\Delta M \approx 0$, then it does not break isospin symmetry. From the above two terms i.e. V_c^{ij} and K.E., and previous work, we can see that I_3 is always conserved but not I^2, and isospin symmetry is broken by the e.m. force and the small mass difference between n and p, and ΔM may arise due to e.m. interactions between nucleons.

We now discuss the effect of CI on isospin invariance for the two nucleon system. Since each nucleon has I=1/2, the two nucleon system can be in I=0 or I=1 states:

$$I=0 : \left(\frac{1}{\sqrt{2}}\right)\{\,|np\rangle - |pn\rangle\,\}, \qquad I_3=0 \qquad (2.32)$$

$$I=1 : \qquad |pp\rangle \qquad\qquad I_3=+1.$$

$$\left(\frac{1}{\sqrt{2}}\right)\{\,|np\rangle + |pn\rangle\,\} \qquad I_3=0 \qquad (2.33)$$

$$|nn\rangle \qquad I_3=-1.$$

where eqn.(2.32) is antisymmetric under interchange of the two nucleons and eqn.(2.33) is symmetric under interchange of the two nucleons.

Thus if we consider the two nucleon system in a state of relative orbital angular momentum L=0 (so that the spatial wave function is symmetric), the generalized Pauli princible {namely, that the total wave function involving isospin, spin and space parts must be antisymmetric for a two fermion system} requires that the I=0 two nucleon state must be in a spin triplet S=1 symmetric state, and the I=1 two nucleon state must be in a spin singlet S=0 antisymmetric state.

Hence isospin invariance or CI requires that the $^1S_0$ N-N interaction must be the same for the three members of the I=1 triplet listed above, i.e.

$$V_{pp}(^1S_0) = V_{nn}(^1S_0) = V_{np}(^1S_0)\ . \qquad (2.34)$$

This can be tested by measuring the $^1S_0$ phase shift δ_0 at very low energies as the above relation implies that:

$$a_{pp} = a_{nn} = a_{np}, \tag{2.35}$$

where the $^1S_0$ scattering length a is defined by

$$a = - \lim_{k \to 0} \left[\frac{\sin\delta_0}{k} \right] \simeq - \lim_{k \to 0} \left[\frac{\delta_0}{k} \right] \tag{2.36}$$

More details about the scattering length are given in section 2.5.

We now discuss charge symmetry, CS, which is the operation of reversing the sign of the 3rd component of isospin. The operator performing this reflection is

$$R = \exp(i\pi I_2) \tag{2.37}$$

where I_2 is the generator of rotations about the second axis in isospin space. Its effect on states is to transform them into states with the opposite sign of I_3. Thus if we apply the above CS operator to I=1, |NN > states, we have

$$\underset{\sim}{R}|NN\rangle_{I_3=+1} = |NN\rangle_{I_3=-1} \qquad (2.38)$$

$$\underset{\sim}{R}|NN\rangle_{I_3=0} = |NN\rangle_{I_3=0} \qquad (2.39)$$

implying under R

$$|pp\rangle \leftrightarrow |nn\rangle$$

Hence CS invariance of the $^1S_0$ NN interaction requires that

$$V_{pp}(^1S_0) = V_{nn}(^1S_0)$$

or in terms of the $^1S_0$ scattering length

$$a_{pp} = a_{nn} .$$

Thus CS invariance is less restrictive than CI invariance.

2.3-CHARGE INDEPENDENCE AND 0^--MESON NUCLEON COUPLING CONSTANT

From $SU_I(2)$ symmetry, relations between coupling constants of different N-N interactions can be found.

Consider the Feynman diagram of Fig.1 which gives rise to the N-N interaction through pion exchange :

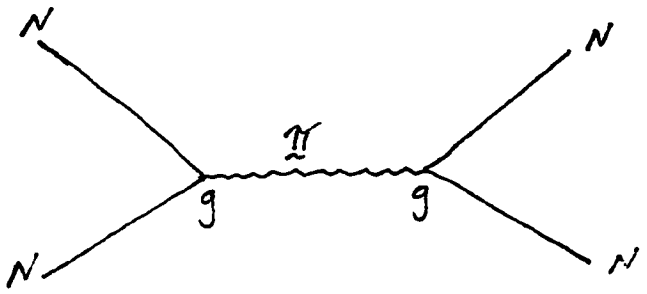

Fig.1

Feynman diagram for exchanged pion.

From Fig.1, at each vertex the $SU_I(2)$ invariant interaction is

$$H_{int} = g\,\bar{N}\;\underline{\tau}\cdot\underline{\pi}\;N, \qquad (2.40)$$

where g is the strength of the vertex and

$$\bar{N} = (\bar{p} \quad \bar{n}) , \qquad N = \begin{bmatrix} p \\ \\ n \end{bmatrix} . \tag{2.41}$$

Here $\bar{n},\bar{p}$ and (n,p) are creation (destruction) operators for nucleon states and $\underline{\pi}$, which mediates the nuclear force between nucleons, is an isospin vector representing three pion states $\pi^{\pm},\pi^{0}$ as follows:

$$\pi^{\mp} \; |0\rangle = \left|\frac{(\pi_1 \pm i\pi_2)}{\sqrt{2}}\right\rangle \tag{2.42}$$

$$\pi^{0} \; |0\rangle = |\pi^{0}\rangle,$$

where $|0\rangle$ is the vacuum state , and $\pi^{\mp},\pi^{0}$ are operators which create $\pi^{\pm}$ and π^{0} mesons respectively. Since $\underline{\pi}$ is an isospin vector, and the only isospin vector which acts on nucleon states available to us is $\underline{\tau}$, we can form an isospin scalar to have $SU_I(2)$ invariance for the pion-nucleon vector as follows:

$$\underline{\tau}.\underline{\pi} = \tau_1 \pi_1 + \tau_2 \pi_2 + \tau_3 \pi_3$$

giving us the expression (2.40). We can now write

$$H_{int} = g \bar{N} [\sqrt{2} \; \tau_{+}\pi^{+} + \sqrt{2} \; \tau_{-}\pi^{-} + \tau_3\pi^{0}]N \tag{2.43}$$

$$H_{int} = g\,[\sqrt{2}\; pn\pi^{+} + \sqrt{2}\; np\pi^{-} + pp\pi^{0} - nn\pi^{0}], \qquad (2.44)$$

where the matrix form of the isospin operator τ is as follows:

$$\tau_{+} = (\tfrac{1}{2})(\tau_1 + i\,\tau_2), \qquad \tau_{-} = (\tfrac{1}{2})(\tau_1 - i\,\tau_2)$$

$$\tau_{+} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad \tau_{-} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \qquad \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \qquad (2.45)$$

Then isospin invariance or CI gives {c.f.Eq.(2.44)}

$$g_{np\pi^-} = g_{pn\pi^+} = \sqrt{2}\; g_{pp\pi^0} = -\sqrt{2}\; g_{nn\pi^0} \qquad (2.46)$$

To find relations between coupling constants of other pseudoscalar 0^- mesons one must consider SU(3) symmetry. The matrix elements of an octet operator are given by

$$\langle 8,k \mid O_i \mid 8,\, j\rangle = i\, f_{ijk}\, F + d_{ijk}\, D \qquad (2.47)$$

where O_i is the octet operator, and $|8,j\rangle$ is the baryon octet state. If the octect operator O_i is taken to be 0^-

pseudoscalar mesons, then the coupling constants are given by

$$g_{ijk} = 2g\,[\, i\, f_{ijk}\, F + d_{ijk} D], \qquad (2.48)$$

where 2g is a normalizing factor so that

$$g_{pp\pi^0} = g[F+D] \ . \qquad (2.49)$$

F and D are the anti-symmetric and symmetric reduced matrix elements respectively. For the case of $g_{pp\pi^0}$, we normalize

$g_{pp\pi^0} = g$, so that

$$F + D = 1 \ . \qquad (2.50)$$

The $\frac{D}{F}$ ratio can be determined experimentally.

We are interested to find $g_{\eta NN}$ and $g_{\eta' NN}$ for our later work [sec.6.2]. In order to do so one has to use Eq.(2.48), we have to relate the physical states $|\eta>$ and $|\eta'>$ to the SU(3) singlet and octet states $|\eta_1>$ and $|\eta_8>$ or to $|\eta_{ns}>$ and $|\eta_s>$, defined as follows (in terms of $q\ \bar{q}$):

$$|\eta_8\rangle = \left(\frac{1}{\sqrt{6}}\right)|\ \bar{u}u+\bar{d}d-\bar{s}s\rangle \qquad (2.51)$$

$$|\eta_1\rangle = \left(\frac{1}{\sqrt{3}}\right)|\ \bar{u}u+\bar{d}d+\bar{s}s\rangle \qquad (2.52)$$

$$|\eta_{ns}\rangle = \left(\frac{1}{\sqrt{2}}\right)|\ \bar{u}u+\bar{d}d\rangle \qquad (2.53)$$

$$|\eta_s\rangle = |\ \bar{s}s\ \rangle\ . \qquad (2.54)$$

In terms of the above states, $|\eta\rangle$ and $|\eta'\rangle$ are given by

$$|\eta\rangle = \cos\phi\ |\eta_{ns}\rangle - \sin\phi\ |\eta_n\rangle \qquad (2.55)$$

$$|\eta'\rangle = \sin\phi\ |\eta_{ns}\rangle + \cos\phi\ |\eta_n\rangle \qquad (2.56)$$

where ϕ is the mixing angle, and $|\eta\rangle$ and $|\eta'\rangle$ are also related to $|\eta_1\rangle$ and $|\eta_8\rangle$ with the mixing angle θ , as follows:

$$|\eta\rangle = -\sin\theta\ |\eta_1\rangle + \cos\theta\ |\eta_8\rangle \qquad (2.57)$$

$$|\eta'\rangle = \cos\theta\ |\eta_1\rangle + \sin\theta\ |\eta_8\rangle\ . \qquad (2.58)$$

The angles θ and ϕ are related by

$$\phi = \theta + \tan^{-1}\sqrt{2}\ . \qquad (2.59)$$

Note also that $\tan^{-1}\sqrt{2}$ in (2.59) is called the ideal

mixing angle.

Since p contains no strange quark, the quark line rule implies that it is not coupled to η_s so that

$$g_{\eta pp} = \cos\phi \; g_{\eta_{ns} pp} \tag{2.60}$$

$$g_{\eta' pp} = \sin\phi \; g_{\eta_{ns} pp} \; . \tag{2.61}$$

Now using Eq.(2.48), we obtain

$$g_{\eta_{ns} pp} = g\,(3F-D) = g_{\eta nn} \tag{2.62}$$

so that Eqs.(2.60)and (2.61) give

$$g_{\eta pp} = g(3F-D)\cos\phi = g_{\eta nn} \tag{2.63}$$

and

$$g_{\eta' pp} = g(3F-D)\sin\phi = g_{\eta' nn} \tag{2.64}$$

and thus

$$\frac{g_{\eta' pp}}{g_{\eta pp}} = \tan\phi = \frac{g_{\eta' nn}}{g_{\eta nn}} \; . \tag{2.65}$$

2.4-CHARGE INDEPENDENT ONE PION EXCHANGE POTENTIAL

One can derive the one pion exchange potential OPEP, which is relevant for the long range part of the N-N potential, from considering the following diagram:

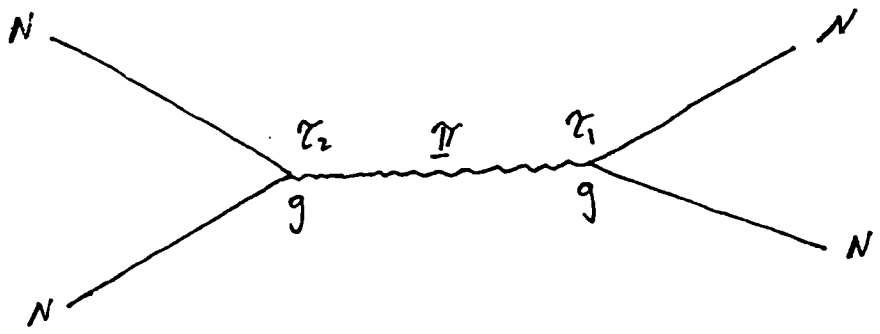

Fig.2

Feynman diagram for exchanged pion with isospin and srenth at each vertix.

As is well known, the above diagram gives the N-N potential

$$V(r) = -\left[\frac{g_r \mu}{2m}\right]^2 \left[\frac{\exp(-\mu r)}{r}\right]\left[-\frac{4}{3}\vec{S}_1\cdot\vec{S}_2+\frac{4}{3}\left[\vec{S}_1\cdot\vec{S}_2 - 3\frac{(\vec{S}_1\cdot\vec{r}\ \vec{S}_2\cdot\vec{r})}{r^2}\right]\right.$$

$$\left.\times\left[1+\frac{3}{(\mu r)}+\frac{3}{(\mu r)^2}\right]\right] \quad . \qquad (2.66)$$

In isospin space, using isospin invariance, this

potential can be written as

$$\bar{V}(r) = \vec{\tau}_1 . \vec{\tau}_2 \; V(r), \qquad (2.67)$$

where $(1/2)\ \vec{\tau}_1$ and $(1/2)\ \vec{\tau}_2$ refer to the isospin of nucleon 1 and nucleon 2.

In Eq.(2.66), m is the common nucleon mass, $\vec{S}_1 = (1/2)\ \vec{\sigma}_1$ is the spin of nucleon 1 and $\vec{S}_2 = (1/2)\ \vec{\sigma}_2$ is the spin of nucleon 2. For the two nucleon system I=1 ($\vec{\tau}_1 . \vec{\tau}_2 = 1$) or I=0 ($\vec{\tau}_1 . \vec{\tau}_2 = -3$). For a system of two nucleons in a state of relative angular momentum L = 0, the generalized Pauli principle requires that I=1, S=0 or I=0, S=1 where S is the total spin of the two nucleon system. Thus the CI potential for the I=1 two nucleon system, namely pp, nn, and $(\frac{1}{\sqrt{2}})\ |np+pn\rangle$ in the $^1S_0$ state is obtained from Eqs.(2.66) and (2.67)

$$V(r) = -\left[\frac{g_r \mu}{2m}\right]^2 \left[\frac{\exp(-\mu r)}{r}\right] \qquad (2.68)$$

which is also known as the Yukawa potential. Here μ is the pion mass. The strength is given by $g^2_r = \frac{g^2}{4\pi}$. For convenience we shall often use $f^2 = (\frac{\mu}{2m})^2\ g^2_r$.

2.5 EFFECTIVE RANGE THEORY

For the scattering of two nucleons at very low energies only the S-wave is important. This can be seen easily from the semi-classical approximation

$$L = Pr \ , \tag{2.69}$$

where L is the angular momentum, P is the linear momentum, and r is the impact parameter of scattering approximated to be the radius of the nucleon (or the nuclear force range) which is approximately 1.5 fm.

The linear momentum is given by

$$P = \sqrt{2 \ m_N \ E_N} \ , \tag{2.70}$$

where E_N is the total energy, and m_N the average nucleon mass. Making the replacement $L^2 \rightarrow \ell(\ell+1)\hbar^2$ where ℓ is the angular momentum quantum number, Eq.(2.69) implies that

$$\left[\frac{\ell(\ell+1)\hbar^2}{2 \ m_N \ E_N}\right]^{\frac{1}{2}} = r = 1.5 \text{ fm} \tag{2.71}$$

For $\ell = 1$, this gives $E_N \simeq 19$ MeV or E_{cm} (center of mass energy for N-N interaction) $\simeq 9$ MeV. Thus we can conclude that for very low energy $\ell = 0$, i.e. only S-wave is important.

Then the radial Schrodinger equation for L=0 is

$$\left[\frac{d^2}{dr^2} + k^2 \right] X(r) = \frac{2\mu}{\hbar^2} V(r)\, X(r) \quad , \qquad (2.72)$$

where

$k^2 = (2\mu \frac{E^2}{\hbar^2})$, E is the center of mass energy of the system, and $\mu = M/2$ is the reduced nucleon mass. The modified radial function is $X(r) = r\, R(r)$, where R(r) is the radial wave function of the system. The asymptotic form of X(r) i.e. as $r \to \infty$, is given by

$$X(r) \simeq C\ (1/k)\ \sin\ (kr + \delta_0\ ) \ , \qquad (2.73)$$

where C is a constant, and δ_0 is the S-wave phase shift caused by a non-zero potential at small r. For $k \to 0$ writing the wave function as X_0, we have

$$\frac{d^2}{dr^2} X_0(r) = \frac{2\mu}{\hbar^2} V(r)\, X_0(r) \quad , \qquad (2.74)$$

where from Eq.(2.73)

$$X_0(r) \xrightarrow[k\to 0]{r\to\infty} C(1/k)\ [\ \sin(kr).\cos(\delta_0) + \cos(kr).\sin(\delta_0)]\ . \tag{2.75}$$

Now for $k \to 0$, $\delta_0 \to 0$ [but not $\frac{\delta_0}{k}$] and

$$\sin(kr) \simeq kr\ , \quad \cos(kr) \simeq 1\ . \tag{2.76}$$

Thus

$$X_0(r) \xrightarrow[k\to 0]{r\to\infty} C\ (1/k)\ \ [\ kr\ \cot\delta_0 + \sin\delta_0\ ], \tag{2.77}$$

$$X_0 \xrightarrow[k\to 0]{r\to\infty} (\sin\delta_0/k)[\ kr\ \cot\delta_0 + 1\ ]\ . \tag{2.78}$$

Define

$$-a = \lim_{k\to 0} \frac{\sin\ \delta_0}{k} \simeq \lim_{k\to 0} \frac{\delta_0}{k}\ , \tag{2.79}$$

where a is called the scattering length. Note that the magnitude of a depends on the strength of the N-N potential since δ_0 depends on the strength of the N-N potential.

Thus finally

$$X_0(r) \underset{r\to\infty}{\simeq} C(-a)\ [-\frac{r}{a} + 1\ ], \qquad (2.80)$$

We can choose $C = -\frac{1}{a}$, so that

$$X_0(r) \underset{r\to\infty}{\simeq} (\ 1 - \frac{r}{a}\ ) \equiv \phi_0(r) \qquad (2.81)$$

where now ϕ_0 is the asymptotic solution of (2.74) i.e.

$$\frac{d^2}{dr^2}\phi_0(r) = 0 \qquad (2.82)$$

since for $r \to \infty$, $V(r) \to 0$.

Now multiply Eq.(2.72) by $X_0(r)$ and (2.74) by $X(r)$ and subtract to get

$$[\ X_0\frac{d^2X}{dr^2} - X\frac{d^2X_0}{dr^2}\ ] = -\ k^2\ X(r)\ X_0(r)\ , \qquad (2.83)$$

i.e.

$$\frac{d}{dr}\ [\ X_0\frac{dX}{dr} - X\frac{dX_0}{dr}\ ] = -\ k^2\ X(r)\ X_0(r)\ .$$

Since $X(0) = X_0(0) = 0$, integration from 0 to r' gives

$$\left[X_0 \frac{dX}{dr} - X \frac{dX_0}{dr} \right] \Bigg|_{r=r'} = - k^2 \int_0^{r'} X(r)\, X_0(r)\, dr \qquad (2.84)$$

Similarly the same steps can be carried out for the asymptotic solutions $\phi(r)$ and $\phi_0(r)$ where

$$X(r) \xrightarrow[r\to\infty]{} \phi(r) \qquad (2.85)$$

and

$$\frac{d^2\phi}{dr^2} + k^2\phi = 0 \qquad (2.86)$$

$$\frac{d^2\phi_0}{dr^2} = 0 \quad . \qquad (2.87)$$

Thus we get

$$\left[\phi_0 \frac{d\phi}{dr} - \phi \frac{d\phi_0}{dr}\right] \Bigg|_{r=r'} - \left[\phi_0 \frac{d\phi}{dr} - \phi \frac{d\phi_0}{dr}\right] \Bigg|_{r=0} = -k^2 \int_0^{r'} \phi\, \phi_0\, dr \quad . \qquad (2.88)$$

Note that $\phi(0)$ and $\phi_0(0) \neq 0$ and from Eqs(2.73) and

(2.85) with $C=-\frac{1}{a}$,

$$\phi_0(0) = \phi(0) = 1, \qquad \frac{d\phi_0}{dr} = -\frac{1}{a}, \tag{2.89}$$

$$\phi(r) = -\frac{1}{ka}\sin(kr+\delta_0) \quad .$$

Let $r\to\infty$, and subtract (2.88) from (2.84) and use Eq(2.85) to get

$$\left[\ \phi_0\frac{d\phi}{dr} - \phi\frac{d\phi_0}{dr}\ \right]\Bigg|_{r=0} = -k^2\int_0^\infty(\phi\ \phi_0 - X\ X_0)\ dr \tag{2.90}$$

Then using Eqs(2.89), it is easy to see that we have

$$[1(-\frac{1}{a})\cos\delta_0 - (\frac{-\sin\delta_0}{ka})(-\frac{1}{a})] = k^2\int_0^\infty(\phi\phi_0 - XX_0)dr, \tag{2.91}$$

or

$$k\cot\delta_0 = -\frac{1}{a} + \frac{1}{2}k^2\ r_0(k) \tag{2.92}$$

where

$$r_0(k) = \left[\frac{-ka}{\sin\delta_0}\right] 2 \int_0^\infty (\phi\ \phi_0 - X\ X_0)\ dr \quad . \qquad (2.93)$$

This is an exact result.

If k is small then to leading order in k, $\frac{\sin\delta_0}{k} = -a$,

and we can write Eq.(2.93) as

$$r_0 = 2 \int_0^\infty (\phi\phi_0 - XX_0)\ dr \quad , \qquad (2.94)$$

where r_0 is now independent of k. Eq.(2.92) then becomes

$$k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} k^2 r_0 \quad . \qquad (2.95)$$

r_0 is called the effective range and is a characteristic of the potential V(r), since $X_0(r)$ is determined by this potential.

Thus we see that low energy nucleon-nucleon data can be analyzed in terms of just two parameters, the scattering length a and the effective range r_0 .

2.6 CHANGE IN SCATTERING LENGTH FOR A CHANGE IN POTENTIAL

Consider the radial Schrodinger equation for L=0 for two different potentials V_1 and V_2, but for $k \to 0$, i.e.

$$\frac{d^2X_1}{dr^2} = \frac{2\mu}{\hbar^2} V_1(r)\, X_1(r) \quad , \tag{2.96}$$

$$\frac{d^2X_2}{dr^2} = \frac{2\mu}{\hbar^2} V_2(r)\, X_2(r) \quad . \tag{2.97}$$

Now multiply Eq.(2.96) by X_2 and Eq.(2.97) by X_1 and substract, to get

$$\frac{d}{dr}\left[X_1\frac{dX_2}{dr} - X_2\frac{dX_1}{dr} \right] = - \frac{2\mu}{\hbar^2}(V_2-V_1)X_1X_2 \quad . \tag{2.98}$$

Integrating both sides of (2.98) over r from $0 \to r'$ and noting that $X_1(0) = X_2(0) = 0$, we get

$$\left[X_1\frac{dX_2}{dr} - X_2\frac{dX_1}{dr} \right]_{r=r'} = - \frac{2\mu}{\hbar^2} \int_0^{\infty} (V_2-V_1)\; X_1(r)X_2(r)dr \quad . \tag{2.99}$$

Consider now the asymptotic form of the Schrodinger equation as k→0

$$\frac{d^2\phi_1}{dr^2} = 0 \quad ,$$

$$\frac{d^2\phi_2}{dr^2} = 0 \quad . \qquad (2.100)$$

This is because V_1 and V_2 vanish as $r\to\infty$. Then we have

$$X_1(r) \xrightarrow[r\to\infty]{} \phi_1 = (1-\frac{r}{a_1}) \quad , \qquad (2.101)$$

$$X_2(r) \xrightarrow[r\to\infty]{} \phi_2 = (1-\frac{r}{a_2}) \quad .$$

Now multiply the first of Eqs(2.100) by ϕ_2 and the second by ϕ_1 , substract and integrate from 0 to r' to get

$$\left[\phi_1\frac{d\phi_2}{dr} - \phi_2\frac{d\phi_1}{dr} \right]\Bigg|_{r=r'} - \left[\phi_1\frac{d\phi_2}{dr} - \phi_2\frac{d\phi_1}{dr} \right]\Bigg|_{r=0} = 0 \qquad (2.102)$$

Now from Eqs(2.101)

$$\phi_1(0) = \phi_2(0) = 1 \quad ,$$

$$\frac{d\phi_1(0)}{dr} = -\frac{1}{a_1} \quad ,$$

$$\frac{d\phi_2(0)}{dr} = -\frac{1}{a_2} \quad . \qquad (2.103)$$

Using (2.103) in (2.102), we obtain

$$\left[\phi_1 \frac{d\phi_2}{dr} - \phi_2 \frac{d\phi_1}{dr} \right]_{r=r'} = -\frac{1}{a_1} - \frac{1}{a_1} \quad , \qquad (2.104)$$

Let now $r \to \infty$, subtract Eq(2.104) from (2.99) and make use of

$$X_1 \xrightarrow[r\to\infty]{} \phi_1$$

$$X_2 \xrightarrow[r\to\infty]{} \phi_2 \qquad (2.105)$$

to obtain

$$\left[-\frac{1}{a_1} - \frac{1}{a_1} \right] = \frac{2\mu}{\hbar^2} \int_0^\infty (V_2 - V_1) X_1(r)\, X_2(r)\, dr \qquad (2.106a)$$

or

$$\left[\frac{\Delta a}{a_1 a_2} \right] = \frac{2\mu}{\hbar^2} \int_0^\infty \Delta V\, X_1(r)\, X_2(r)\, dr \quad . \qquad (2.106b)$$

This is an exact result and relates the change in scattering length due to a change in the potential.

CHAPTER THREE

EXPERIMENTAL PARAMETERS

The experimental values of the N-N scattering lengths gave the first suggestion for charge independence. In the case of n-p scattering there is no difficulty in collecting the data needed to find the scattering length, but the bound state effect must be removed. This can be achieved by scattering at an energy E < 10 MeV which is then dominated by scattering in the S=0 state, with np scattering in the S=1 "deuteron" state being energetically unallowed.

In p-p scattering there is a difficulty of removing the effect of the electromagnetic interaction, and this is model dependent in some aspects. For the third case, n-n scattering, it is very difficult to collect data because of the difficulty of getting neutron targets and high intensity neutron beams. We will consider each one of these three cases separately.

3.1- n-p SCATTERING

The n-p scattering data are the most accurate low energy information which exist. As stated earlier, the energy must be below 10 MeV (otherwise contributions from the deuteron must be removed, which is a very difficult process) but there are no long range effects which need to be corrected for. There are other effects which can be overcome [see ref.2 and references therein]. The accepted value for a_{np} is [see ref.1],

$$a_{np} = -23.748 \pm 0.010 \text{ fm} . \qquad (3.1)$$

The effective range is also given by (ref.1)

$$r_{np} = 2.75 \pm 0.05 \text{ fm} . \qquad (3.2)$$

3.2- p-p SCATTERING

The collected data on p-p scattering is very precise because of the charge of the proton, since it is much easier to detect charged particles than uncharged ones.

On the otherhand, it is easier to accelerate protons and one can also have pure targets of protons i.e. liquid hydrogen. The experimental values for a_{pp} and r_{pp} are given by [e.g.ref.2]

$$a_{pp}^{E} = -7.8098 \pm 0.0023 \quad \text{fm} ., \tag{3.3}$$

$$r_{pp}^{E} = 2.767 \pm 0.010 \quad \text{fm} ., \tag{3.4}$$

where E stands for the scattering length uncorrected due to long range interactions like the Coulomb interaction and vacuum polarization. The difficulties with p-p scattering are the corrections due to the Coulomb interaction and vacuum polarization. The scattering length corrected for the Coulomb interaction is given by the somewhat model-dependent formula

$$\frac{1}{a_{pp}} = \frac{1}{a_{pp}^{c}} + \frac{1}{D}\left[\ \ln\{(\frac{D}{r_{pp}}) - 0.33\}\ \right] \tag{3.5}$$

where a_{pp}^{c} is the scattering length in the presence of Coulomb interaction, and

$$D = \frac{1}{Me^{2}} = 28.82 \text{ fm} . \tag{3.6}$$

Eq.(3.5) still has to be corrected for vacuum polarization. For more detail see (ref.2).

The accepted values of a_{pp} and r_{pp} are (from ref.1),

$$a_{pp} = -17.9 \pm 0.002 \text{ fm} ,$$

$$r_{pp} = 2.82 \pm 0.01 \text{ fm} . \qquad (3.7)$$

In (ref.1) the authors did not combine their values with errors, but we will take them to be as indicated above [c.f.ref.6] .

3.3- n-n SCATTERING

There are difficulties in collecting data for n-n scattering. The main difficulty is having a pure neutron target, while the other one is to have a high intensity beam of neutrons. On account of these difficulties, and others, direct n-n scattering is very difficult to carry out.

Because of such difficulties, indirect experiments have been used to measure the n-n scattering length and effective range. Usually three-body final state interactions occur in indirect n-n scattering, where two of the final particles are neutrons. The spectrum of

the third particle is most often measured, where the number of events near the upper end of its kinematical limit is the measure of the low energy n-n interaction. The best third particle is one that does not interact with neutrons, i.e. the interaction of the third particle with n is negligible, otherwise one will face the three body problem which is complicated. If the third particle interacts significantly with neutrons, then the scattering length and effective range depend on having a valid theory of three body interactions. The first suggested indirect reaction is

$$\pi + d \quad \text{-----} > \quad \gamma + n + n \quad , \qquad (3.8)$$

where the γ-n interaction is negligible. The second indirect reaction is

$$n + d \quad \text{-----} > \quad p + n + n \quad . \qquad (3.9)$$

In this reaction the n-p interaction is as strong as n-n and a 3-body analysis must be made. From (ref.1) the scattering length and effective range are given by

$$\left.\begin{aligned} a_{nn} &= -18.45 \pm 0.46 \text{ fm} \\ r_{nn} &= 2.83 \pm 0.16 \text{ fm} \end{aligned}\right\} \quad \text{from } d(\pi^-,\gamma)2n, \qquad (3.10)$$

and

$$a_{nn} = -16.73 \pm 0.47 \text{ fm} \quad \Big\}$$

$$\Big\} \text{ from } d(n,p)2n \,,$$

$$r_{nn} = 2.68 \pm 0.16 \text{ fm} \quad \Big\} \qquad (3.11)$$

The preferred values are those determined by [see ref.1]. the $d(\pi^-,\gamma)2n$ reaction i.e. the values in Eq.(3.10) [For more details see ref.2] .

From the above experimental data, we see that

$$a_{pp} - a_{nn} = 0.55 \pm 0.46 \text{ fm} \qquad (3.12)$$

$$\Delta a = \left\{\frac{(a_{pp} + a_{nn})}{2} - a_{np}\right\}$$

$$= 5.5 \pm 0.3 \text{ fm} \quad . \qquad (3.13)$$

As discussed in chapter two, CS implying vanishing of $\delta a = a_{pp} - a_{nn}$, holds within the experimental error but more precise data is needed to test it. The vanishing of Δa implies CI but the above experimental value of Δa shows that CI is broken. In the next chapter we discuss the various charge dependent effects on NN interactions in order to explain the above value of Δa.

TERMS	EXP.(fm)
a_{np}	-23.748±0.01
a_{nn}	-18.45 ±0.46
a_{pp}	17.9 ±0.002
r_{np}	2.75 ±0.05
r_{nn}	2.83 ±0.16
r_{pp}	2.82 ±0.001

TABLE 3.1

Experimental data (ref.1) for N-N scattering length a and effective range r are given (in fm). The error in a_{pp} is not stated in (ref.1), but was taken from (ref.6).

CHAPTER FOUR

CHARGE DEPENDENT EFFECTS

In this chapter the charge dependent effects on the I=1 $^1S_0$ scattering lengths are considered. The measured values of the scattering lengths a and the effective ranges r_0 are given in table 3.1. Charge symmetry, as we saw earlier, implying vanishing of the difference $a_{pp} - a_{nn}$, holds within the experimental errors since $\delta a \simeq 0.55 \pm 0.46$ fm. Charge independence, which would imply the vanishing of the quantity $\Delta a = [(\frac{a_{pp} + a_{nn}}{2}) - a_{np}]$, is broken by the measured $\Delta a \simeq 5.5 \pm 0.3$ fm .[Note that we used $\bar{a} = \frac{1}{2} \{a_{pp} + a_{nn}\}$ in charge independence (CI) to remove the effect of charge symmetry (CS) from CI]. The understanding of this discrepency in terms of direct electromagnetic effects (other than Coulomb) and charge dependence corrections to the meson masses and coupling constants has a long history summarized in ref.2.

Briefly, charge dependent effects may be broken into two types, direct and indirect electromagnetic effects. The direct e.m. effects, apart from the Coulomb effect which has already been taken care of in quoting the values in table 3.1, include (a)vacuum polarization, (b)the neutron-proton mass difference, and (c) magnetic effects due to the proton-neutron magnetic moment difference and finite size effects due to the charge and magnetic moment distributions of the proton and neutron. The indirect e.m. effects include (a) the $\pi^{\pm} - \pi^{0}$ meson mass difference in the one pion exchange potential (OPEP), (b) the $\pi^{\pm} - \pi^{0}$ meson mass difference in the two pion exchange potential (TPEP), (c) charge dependence of π-N coupling constants, (d) $\pi\gamma$ exchange potential, and (e) up-down quark mass difference. The direct effects result in a small contribution to a, as follows:

$$\begin{aligned} \Delta a_{pp} \text{ (magn. + finite size)} &\sim -0.06 \text{ fm.} \\ \Delta a_{nn} \text{ (magn. + finite size)} &\sim +0.02 \text{ fm.} \\ \Delta a_{np} \text{ (magn. + finite size)} &\sim +0.20 \text{ fm.} \end{aligned} \qquad (4.1)$$

The effect of vacuum polarization and the n-p mass difference are still smaler. Out of the indirect e.m. effects, the most important is due to the $\pi^{\pm} - \pi^{0}$ mass difference.

Now we will consider the indirect effects in some detail. The perturbation expression for Δa is given by Eq.(2.106), i.e.

$$\Delta a = (\bar{a} - a_{np}) = \bar{a}\, a_{np} m \int_0^\infty (\bar{V} - V_{np})\, X\, X_{np}\, dr \,, \qquad (4.2)$$

where $\bar{a}$ stands for the average scattering length [in the case of CI of a_{pp} and a_{nn} this is $\bar{a} = \frac{1}{2}\{a_{nn} + a_{pp}\}]$, a_{np} stands for the n-p scattering length. The $\bar{V}$ potential stands for $\bar{V} = \frac{1}{2}\{ V_{pp} + V_{nn} \}$, and V_{np} denotes the n-p potential. The wave functions X and X_{np} are the zero energy $^1S_0$ wave functions associated with $\bar{V}$ and V_{np} respectively, and normalized so that

$$X(r) \xrightarrow[r\to\infty]{} \left(1 - \frac{r}{a} \right) \,. \qquad (4.3)$$

m is the average nucleon mass. The large factor $\bar{a}\, a_{np}\, m$ on the right hand side of Eq.(4.2) makes Δa very sensitive to even a small change in the potential difference $\Delta V = \bar{V} - V_{np}$.

It is convenient to use experimental values for

$\bar{a}$ and a_{np} such that the perturbation potential ΔV is to result in the experimental values of Δa (see ref. 3). Now, after fixing the value of the factor $\bar{a}a_{np}$ m in Eq.(4.2), Δa is additive since ΔV is additive. The distinction between X(r) and $X_{np}(r)$ is not important, since consideration of this difference will be of second order in ΔV . We will use $|X_0(r)|^2$ insted of XX_{np}. $X_0(r)$ is generated by using the Reid soft core potential (ref. 15). The details of computing the wave function are considered in chapter 5. As stated earlier Δa is additive, allowing us to evaluate the various indirect effects listed earlier separately.

4.1 ELECTROMAGNETIC CORRECTIONS TO COUPLINGS

In this section we will consider the electromagnetic (e.m.) correction to the pion-nucleon coupling constants. This correction is of the order α^2, where α is the fine structure constant $\alpha \approx 1/137$. This correction causes the charged pion couplings to differ from the neutral pion couplings, i.e.

$$f_{\pi^0 p} = f\{1 - \frac{\delta_0}{2}\}$$

$$f_{\pi^0 n} = f\{1 + \frac{\delta_0}{2}\} \qquad (4.4)$$

and

$$f_c = (\frac{1}{2})\{ f_{\pi^- p} + f_{\pi^+ n} \} = f(1+\delta_c) \quad . \qquad (4.5)$$

δ_0 and δ_c are the correction to the couplings when α^2 is considered in calculating the coupling constants from the Feynman diagram, where f is the coupling when all the couplings are the same, i.e. the e.m. second order corrections to couplings are neglected. Also $f_{\pi^- p} \simeq f_{\pi^+ n}$. These corrections have been evaluated by Morrison (ref.10) as

$\delta_0 = 0.0023$, $\delta_c = -0.0058$.

In the next section we consider Feynman diagrams to see the effect of these corrections on OPEP and TPEP.

4.2 POTENTIALS

1 - ONE PION MASS DIFFERENCE EFFECT

The effect of the $\pi^{\pm} - \pi^{0}$ mass difference on OPEP as given in Eq.(2.98), can be derived from Figs.3(a,b,c) and is given by:

$$\Delta V_{CI} = \overline{V} - V_{np} \ ,$$

where from Fig.3(a)

$$\overline{V} \equiv \frac{1}{2}(V_{pp} + V_{nn}) = C(0), \qquad (4.6)$$

and from Fig.3(b)&(c),

$$V_{np} = -C(0) + \sqrt{2} \ . \ \sqrt{2} \ C(1) \ ,$$

so that

$$\Delta V_{CI} = 2 \ [\ C(0) - C(1) \ ] \qquad (4.7)$$

where

$$C(0) = - g^2 \ (\mu_0/2m)^2 \ \{ \ \exp(-\mu_0 r)/r \ \}$$

and

$$C(1) = - g^2 \ (\mu_c/2m)^2 \ \{ \ \exp(-\mu_c r)/r \ \} \ , \qquad (4.8)$$

as taken from Eq(2.106). Thus

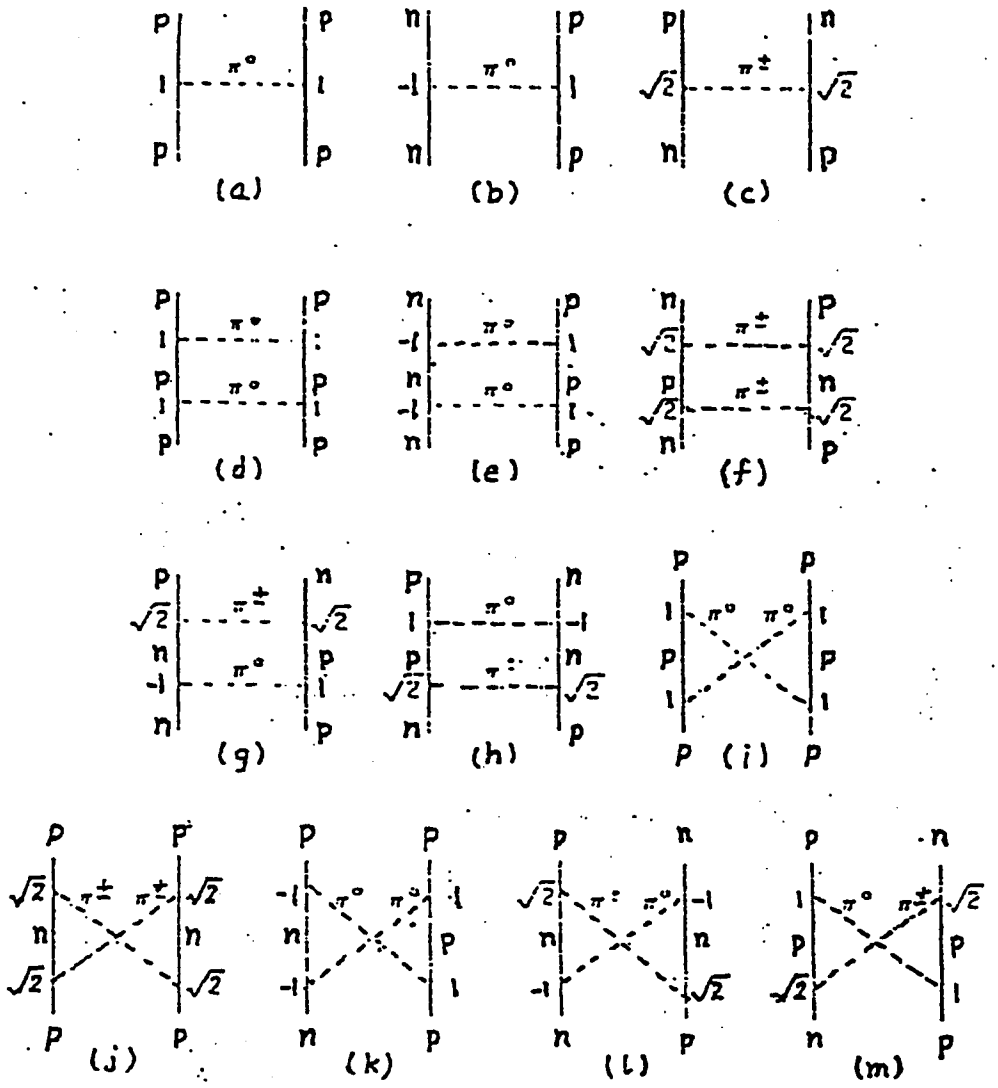

FIGURE 3

Charge dependence of OPEP and TPEP. Graphs (a) to (c) are the three OPEP cases, graphs (d) to (h) represent the uncrossed TPEP digrams, and graphs (i) to (m) represent the crossed TPEP diagrams. The isospin relative coefficients are given at each vertex. These graphs were taken from (ref.12, Fig.1).

$$\Delta V_{CI,\Delta\mu} = -g^2 (\mu_0/\mu_c)^2 \{\exp(-\mu_0 r)/r\}[1-(\mu_c/\mu_0)^2 \times \{ \exp(-\Delta\mu r)/r \}] \quad (4.9)$$

where

μ_0 = mass of π^0

μ_c = mass of $\pi^{\pm}$

$f^2 = g^2(\mu_0/2m)^2$

m = average nucleon mass

$\Delta\mu = \mu_c - \mu_0$

2 - COUPLING CONSTANT EFFECTS IN OPEP

The OPEP due to the e.m. correction can be found by using equ.(4.4) and (4.5) and replacing in Fig.3(a,b,c) the factors 1 , -1 and $\sqrt{2}$ respectively by $(1-\delta_0/2)$, $-(1+\delta_0/2)$ and $(1+\delta_c)$, and neglecting terms of second order in the δ's. Thus V_{np} becomes, from Fig.3(c),

$$V_{np} = -C(0)+\sqrt{2}\ (1+\delta_c)\ \sqrt{2}\ (1+\delta_c\ )\ C(1)\ . \quad (4.10a)$$

Note that there will be no correction in C(0) due to δ_c but there will be one due to δ_0 but it is of the order of (δ_0^2) which is very small. Thus finally

$$\Delta V^{OPEP}_{CI,\Delta\mu+cc}=2[C(0)-C(1)]\ -4\delta_c C(1)+O(\delta_c^2)+O(\delta_0^2), \qquad (4.10b)$$

where the effect of δ_0 is of higher order as can be seen from

$$V_{np} = -(1+\frac{\delta_0}{2})(1-\frac{\delta_0}{2})\ C(0) + 2(1+\delta_c)^2\ C(1) \qquad (4.11a)$$

$$\bar{V} = \frac{1}{2}[(1+\frac{\delta_0}{2})^2 + (1-\frac{\delta_0}{2})^2]\ C(0) = \frac{1}{2}[2+\frac{\delta_0^2}{2}]\ C(0) \qquad (4.11b)$$

so that

$$\Delta V^{OPEP}_{CI} = 2\ C(0) + O(\delta_0^2). \qquad (4.11c)$$

This implyies for OPEP [c.f.Eq.(4.7)] that

$$\Delta V^{CI}_{cc} \simeq 4\ \delta_c\ f^2[\ \exp(-\mu r)/r] \qquad (4.12)$$

where "cc" denotes correction due to coupling constants.

3 - TWO PION MASS DIFFERENCE EFFECT

In the case of TPEP, Figs.3{d,...m} are used to write down the potentials, and we denote the amplitude of the uncrossed diagrams {d,...h} by A, and those for the crossed diagrams {i,...m} by B. Each amplitude is assigned two indices, which denote the kind of exchanged π - meson (i.e. 0 & 1 for π^0 & $\pi^\pm$ respectively). In

this way, we obtain for the effect of the $\pi^{\pm}-\pi^{0}$ mass difference on TPEP :

$$V = A(00) + B(00) + 4\,B(11),$$

$$V_{np} = A(00) + 4A(11) - 4\,A(01) + B(00) + 4\,B(01) \qquad (4.13)$$

implying

$$\Delta V^{TPEP}_{CI,\Delta\mu} = 4[\{\,B(11) - A(11)\,\} - \{\,B(01) - A(01)\,\}] \qquad (4.14)$$

4 - COUPLING CONSTANT EFFECT IN TPEP

The coupling constant's correction to the TPEP $\Delta V_{CI,cc'}$ can be obtained from Fig.3(d,----m) by replacing the factors 1 , -1 and $\sqrt{2}$ in the appropriate Feynman diagrams respectively by $(1-\frac{\delta_0}{2})$, $-(1+\frac{\delta_0}{2})$ and $(1+\delta_c)$ and neglecting terms of second order in the δ's. This term is given by

$$\Delta V^{TPEP}_{CI,cc} \approx 8\,\delta_c[\,B(11) - A(11)\,]\,, \qquad (4.15)$$

For more details of the derivation of Eqs (4.14) and (4.15) and the complete form of these potentials see Appendix A.

The last CI term is the $\pi\gamma$ exchange potential and its value has been evaluated to be $\Delta a_{\pi\gamma} = 1.1$ fm (ref. 3).

4.3-CHARGE SYMMETRY BREAKING TERM DUE TO ELECTROMAGNETIC CORRECTION

The charge symmetry breaking potential is given by $\Delta V_{CS} = V_{pp} - V_{nn}$. This expression is to be evaluated for the OPEP and TPEP due to the e.m. correction of the coupling constants.

In the case of OPEP there is no $\pi^{\pm} - \pi^{0}$ mass difference correction to ΔV_{CS} and thus we consider the effect due to the e.m. correction of the pion-nucleon coupling constants. Using equ.(4.4) and Fig.3(a), we have,

$$V_{pp} = (1 - \frac{\delta_0}{2})^2 \quad C(0)$$

$$V_{nn} = (1 + \frac{\delta_0}{2})^2 \quad C(0) \qquad (4.16)$$

so that

$$\Delta V_{CSB}^{OPEP} = \left[\left(1-\delta_0+\frac{\delta_0^2}{4}\right) - \left(1+\delta_0+\frac{\delta_0^2}{4}\right) \right] C(0)$$

$$\simeq -2\,\delta_0\, C(0) \qquad (4.17)$$

[neglecting second order term in δ_0^2]

and thus

$$\Delta V_{CSB}^{OPEP} = 2\delta_0\, f^2[\, \exp(-\mu_0 r)/r \,] \,. \qquad (4.18)$$

In the case of TPEP due to the e.m. correction, using Fig.3{d,...m} and Eq.(4.4) we get:

$$\Delta V_{CSB}^{TPEP} = -4\,\delta_0[\, A(00) + B(00) \,] \,. \qquad (4.19)$$

For the derivation of (4.19) see Appendix B.

The contributions to Δa from various potentials are summarised in table 4.1 , and the shapes of the integrands are shown in Figs. 4 and 5.

TERMS	Δa(fm)
$\Delta a_{\Delta\mu}^{OPEP}$	3.6 ± 0.15
$\Delta a_{\Delta\mu}^{TPEP}$	0.88 ± 0.1
Δa_{cc}^{OPEP}	-1.6 ± 0.2
Δa_{cc}^{TPEP}	+1.6 ± 0.2
$\Delta a_{\pi\gamma}$	1.1 ± 0.5
Δa_{tot}	5.6 ± 0.5

TABLE 4.1

Contribution to Δa from different potentials discussed in this chapter. The errors were taken from (ref.3). For more details of how these errors are evaluated see (ref.3).

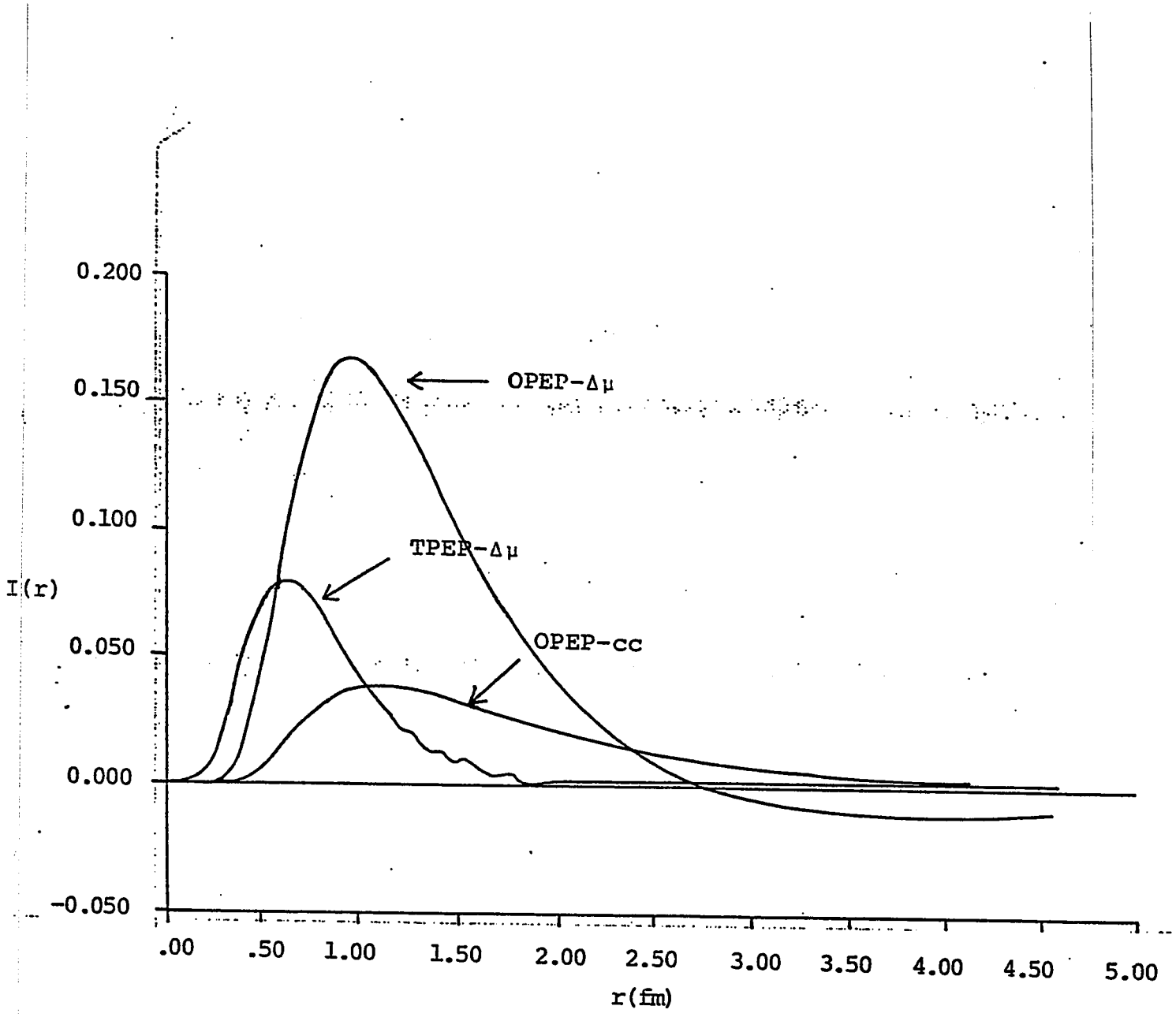

FIGURE 4

Integrands of Eqn.(4.1), for the indicated potentials.

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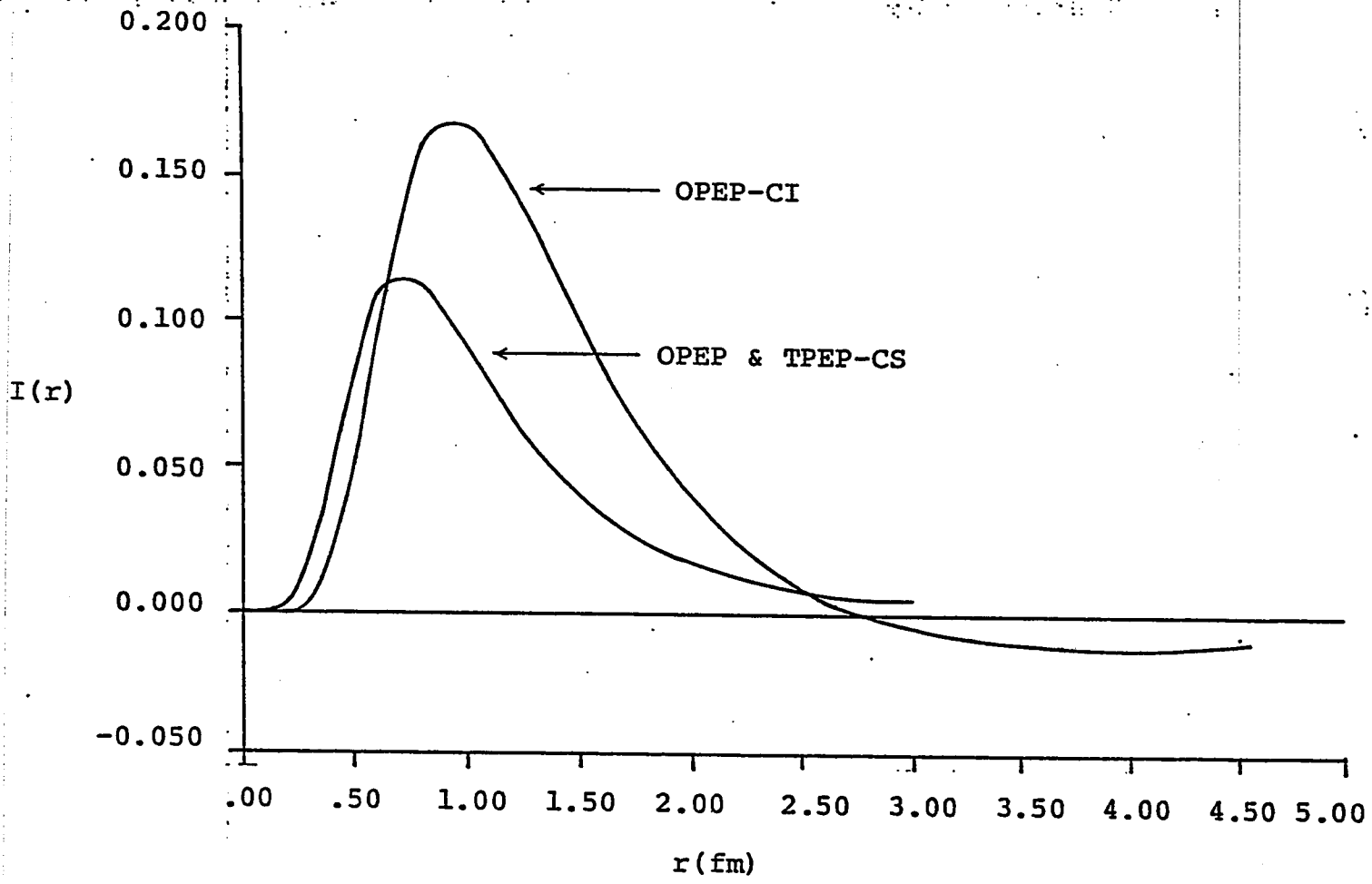

FIGURE 5

Integrands of Eqn.(4.1), for the indicated potentials.

CHAPTER FIVE

COMPUTATION

In this chapter we will consider the computational techniques used to get the results of this thesis. The first section is an introducton for this chapter which gives an overview and reasons for considering this chapter. In the second section, we will consider evaluation of the wave function. In section three, the ways of calculating the OPEP and OBEP contributions are considered. In section four, TPEP will be considered in more detail. The final section will contain numerical values of the physical constants used in this thesis.

5.1 - INTRODUCTION

In our work we needed evaluation of quantities which were very senstive to small changes in potentials, for example the CIB and CSB TPEP expressions, so accurate programmes were needed which could do this job. The most difficult potential - in these calculations, the TPEP, was calculated but unfortunately it was not exactly the same as what Ericson and Miller (ref.3) computed. During the work, it was found many times that it was needed - for self training and confidence - to numerically compute work which had previously been done. Unfortunately, some times it was not easy to reproduce such results. The missing point in the above statement was that there was almost not any written programme included with the results stated. For this reason it is better to include some of the programmes used in this thesis where it is neccessary to include them. This will be done in Appendix C.

5.2 - N-N WAVE FUNCTION

We need the N-N wave function u(r) in the perturbative expression Eq.(4.1) so that one can evaluate the different contributions to CI and CS. To find such a wave function one has to solve the Schrodinger equation i.e.

$$\frac{d^2}{dr^2} u(r) = \frac{2\mu'}{h^2} V(r)u(r) \qquad (5.1)$$

where μ' is the reduced nucleon mass and we have taken $k \to 0$ and $L=0$ as in the case for very low energy scattering.

Since V(r) is not well known, and even the approximate potential is not known in terms of integrable expressions, we can not find an analytic solution for the wave function. One way to get the wave function is by solving eq(5.1) numerically. In this section we will use the same notation as in ref.(15). For N-N scattering $2\mu' = m$ = average nucleon mass. To have the same notation as in ref.15, divide Eq(5.1) by the pion-mass and let $x=\mu r$, then we will get

$$\frac{d^2}{dx^2}\, u(r) = V'(r)u(r) \qquad (5.2)$$

where

$$V'(r) = \left(\frac{m}{\mu^2\, h^2} \right) V(r)$$

$\mu = \left(\frac{m_0 c}{h}\right) \simeq 0.7$ fm. (we use natural units)

A $^1S_0$ I=1 phenomenological N-N potential, called the Reid soft-core potential, is given by ref.(15). Its form is;

$$V(x) = -h\frac{\exp(-x)}{x} - 1650.6\frac{\exp(-4x)}{x} + 6484.2\frac{\exp(-7x)}{x}, \qquad (5.3)$$

where h=10.463 MeV ,and the unit of the potential is in MeV.

The asymptotic solution of the wave function is

$$u(r) \xrightarrow[k\to 0]{} \left(1 - \frac{r}{a} \right) . \qquad (5.4)$$

We used the programme listed at the end of this thesis to find the solution of the second order differential equation. The boundary conditions needed to solve this differential equation are

$$u(0) = 0 ,$$

$$u'(0) = \frac{du(0)}{dr} \neq 0 \qquad (5.5)$$

and the asymptotic solution (5.4). We used an arbitrary value for u'(0) at r=0 and then normalized the wavefunction by matching it to the asymptotic solution (5.4). Integration was done from x = 0.035 to 3.22, with a step size of $\Delta x = 0.35$ and error tolerance of 0.0001, implying 0.01 percent is the maximum error tolerated. From x = 3.22 to 10.0, we used (5.4) to find u(r) since V(r) is negligible beyond $x = \mu r = 3.22$, i.e. for $r \geq 4.6$ fm. Actually we followed the same method used by ref.3 as it appears to be from Fig.1 of ref.3. Then we matched the asymptotic solution with the small r solution at R=4.6 fm to get the shape in Fig.6. The final comment to mention here is that we used

$$a^{exp.} = -18.2 \text{ fm} = (\frac{1}{2})\{ a_{pp} + a_{nn}\},$$

in (5.4), since there is no difference in using any of the experimental scattering lengths in evaluating Eq.(5.4), and this is due to the insensitivity of the wave function to the scattering length.

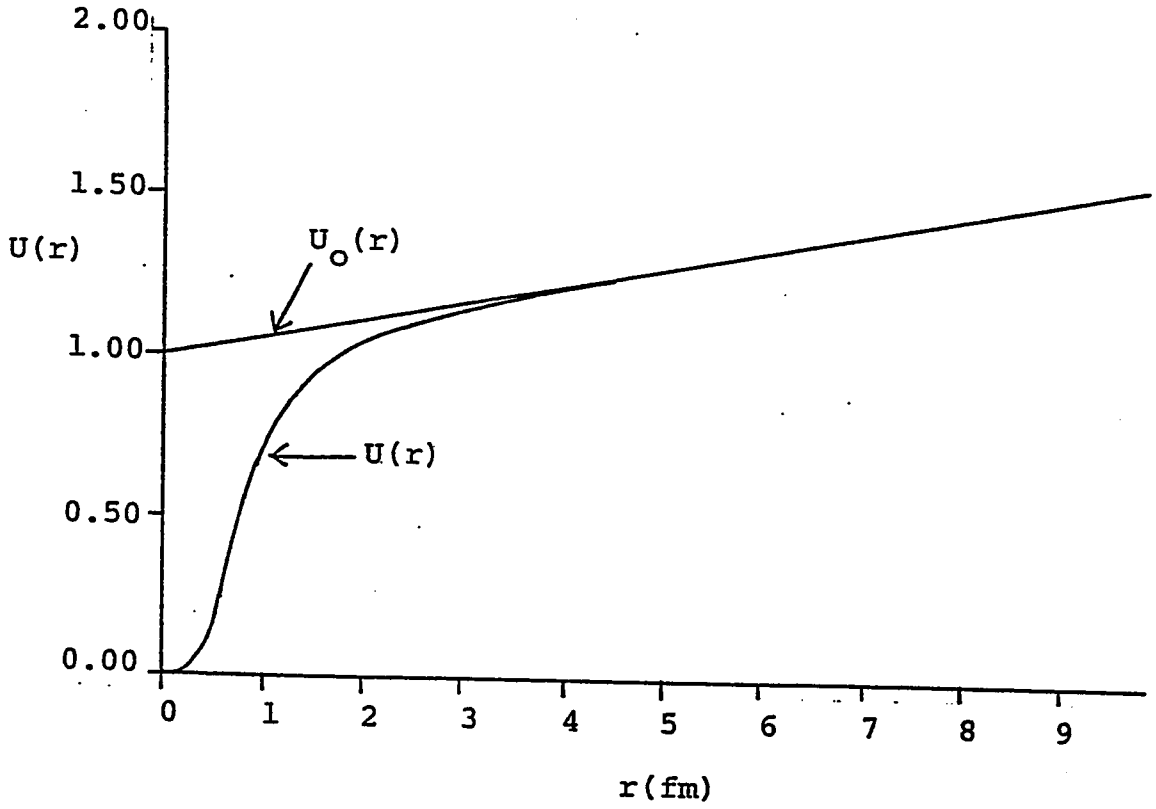

FIGURE 6

The N-N wave function u(r) and the asymptotic solution $u_0(r)$ at very low energy.

5.3-EVALUATION OF ONE BOSON EXCHANGE POTENTIAL (OBEP) CONTRIBUTION

Since we know each expression of OBEP [i.e.Eqn.'s(4.9,12,18,and 6.33)], there is no difficulty in evaluating these expressions. There is one restriction in evaluating these terms, i.e. the potentials must be evaluated at the same radial points as the wave function. This solution will make one loose error control. For this reason I have checked the error for some similar known functions and found that for the interval length Δr = 0.05 fm the maximum error in a case was 7.0 percent. This restriction is not limited to OBEP only, but extended to TPEP also.

5.4-EVALUATION OF TPEP

The most difficult and time consuming potential was the valuation of TPEP. The expressions of TPEP are given in ref.(12), and each potential is formed out of more than two terms and those terms are either two or three dimensional integrals. Since, as we mentioned in chapter four, the perturbative expression [Eq(4.1)] being additive allows us to evaluate each term separately. Thus we need to evaluate TPEP first and then put it in Eq(4.1) to find Δa^{TPEP} . In ref.(12), they advised to use appendix B of their paper to evaluate the different terms. I used their advice but, unfortunately, I got ΔV^{TPEP} about 20 times larger than the actual potential difference in the case of CS. I think the reason behind this is that a lot of error is introduced during computation, specially that we have multi-dimensional integrals, and also we have a large number of terms to be added or subtracted many times until we get the last required value of ΔV^{TPEP} .

The second way I used to evaluate these terms is by using a programme that controls error. The subroutine QB01A from the Harwell Subroutine Library ref.(17),

which controls the error during integration, was used to evaluate those potentials. Note that in using QB01A, I used the transformation suggested by ref.(12), in their appendix B, but I did not use 21 points at each dimension. In this case I also faced difficulties, the first one was that the ΔV_{CS}^{TPEP} potential difference changes sign at some values of r, while it should not, the second difficulty was the long time needed to run each programme (not CPU time) . When the error tolerance is specified by 10^{-5}, it took about 24 hours to run the programme because of the three dimensional integrations and because of the small error tolerance. The last method I used was to have an error tolerance of 10^{-4} and then I got the shape in Fig.4.

Still there is some problem in the advice of ref.(12) with respect to the limits of integration. When we followed the advice of ref.(12) and changed variables to expand the domain of integration and to get the required result, we got a wrong shape. The important point for integration, i.e. to get the correct result, is by unifying the limits before changing the variables and to make the change of limit according to the change of variables. Table (5.1), shows the change of limits. Normally the limits of the integrations can be written

from 0 to 1, but during computation we can not use 0 or 1, otherwise there will be singularities. To scan all the domain of integration and not to loose any important contributiuon to the potentials, we chose very close values to the limits, i.e. the variables before change have the following values, [c.f.App.A and B].

$$\beta_L = \zeta_L = \xi_L^{-1} = 10^{-12}$$

$$\beta_u = \zeta_u = \xi_L^{-1} = 0.99$$

where L stands for the lower limit and u for the upper limit.

TABLE 5.1

This table shows the transformation of variables and limits for the expressions listed in appendices A and B and according to appendix B of ref.12.

TERM	VARIABLE	UPPER LIMIT	LOWER LIMIT	COMMENTS
all terms	β	0.99	10^{-12}	no change of variables
X's	$u = \zeta^{\frac{1}{3}}$	0.99	10^{-4}	variables did change
Y's	$u = \zeta^{\frac{1}{6}}$	0.99	0.01	variables did change
W	$u = \zeta^{\frac{1}{3}}$	0.996655	10^{-4}	variables did change
	$w = \xi^{-\frac{1}{3}}$	0.99	10^{-4}	
Z	$u = \zeta^{\frac{1}{6}}$	0.996655	0.01	variables did change
	$w = \xi^{-\frac{1}{3}}$	0.99	10^{-4}	

TABLE 5.2

This table contains all the constants used in this thesis. Note that natural units are used in this table, i.e. h = c = 1, except for hc.

SYMBOL	VALUE	NAME
$\mu^{\pm}$	139.6 MeV/c	charged pion mass
μ^{0}	135.0 MeV/c	neutral pion mass
m	938.906 MeV/c	avearge nucleon mass
η	548.8 MeV/c	η-particle mass
η'	958.0 MeV/c	η'-particle mass
f^2	0.08	unit less coupling constant
hc	197.33 MeV fm	in natural unit they equal to unity
K	493 MeV/c	K-meson mass
m_u m_d m_s	360 MeV/c 360 MeV/c 510 MeV/c	up , down and strange constituent quark mass
m_u m_d	5 MeV/c 10 MeV/c	up and down current quark mass

CHAPTER SIX

CHARGE SYMMETRY BREAKING CORRECTIONS TO NN SCATTERING LENGTH DUE TO UP-DOWN CURRENT QUARK MASS DIFFERENCE

In this chapter, the charge symmetry breaking (CSB) correction due to the isospin breaking part of the QCD Hamiltonian, determined by the up-down current quark mas difference $H_{QCD} = (1/2)\,(\,m_u - m_d\,)(\,\bar{u}u - \bar{d}d\,)$, on $(\delta a)_{CBS} = a_{pp} - a_{nn}$ is estimated. This is done by first calculating the effect of H_{QCD} on the pion-nucleon coupling constants via π^0-η,η' mixing, giving rise to CSB one pion exchange (OPEP) and two pion exchange (TPEP) potentials. The importance of CSB-TPEP is emphasized. The η-nucleon coupling constant appears to be constrained by $(\delta a)^{exp.}_{CSB}$ to be very small.

6.1 - INTRODUCTION

As seen in chapter 4, $(\Delta a)_{CIB}$ is almost entirely explained by the various corrections listed in Table-4.1, the most important of these corrections is the $\pi^{\pm} - \pi^{0}$ mass difference. The situation with respect to CSB effects is, however, not so clear as it is in CI. Apart from CSB, OPEP and TPEP potentials (ref.3) generated by the e.m. corrections to the $pp\pi^{0}$ and $nn\pi^{0}$ coupling constants, there is an additional effect due to the difference between the masses of the up and down quarks. Although such a mass difference is regarded as of non-electromagnetic origin and is believed to arise from QCD and the Higgs mechanism, it does violate isospin symmetry. Various authors (refs. 4,5,6 & 7) have estimated the effect of such a mass difference on the scattering lengths and have generally found it to be small. In ref.4 the effect of $\Delta m_q = (m_u - m_d)$ has been investigated on the pion nucleon coupling constants within the framework of the MIT bag model. The effect of Δm_q on an effective nucleon-nucleon potential was studied in ref.5 in a quark cluster model. A related investigation has been done in ref.6 where the so called color magnetic correction to the short range nucleon-nucleon potential arising from the one gluon exchange quark-quark

potential has been calculated. The isospin violating effect on the scattering lengths in such a calculation also arises from Δm_q. A more recent calculation has studied CSB in OPEP arising from the small difference in the charge RMS radii for u- and d-quark distributions in a nucleon due to Δm_q (ref.7). A common feature of all these calculations is that they need the up-down constituent quark mass difference rather than the current quark mass difference. On the other hand it is for the latter that a reliable estimate exists: $\Delta m_q \simeq 5$ MeV (with current quark masses being $m_u \simeq 5$ MeV, $m_d \simeq 10$ MeV). Thus in the above mentioned calculations one has to rely on the expectation that the mass difference $\Delta m_q \simeq 5$ MeV is not lost in the renormalization of the current quark masses to those of the constituent quarks (notice that the common constituent quark mass is 360 MeV). The approach we wish to follow is to estimate the effect of Δm_q on the pion-nucleon coupling constants where Δm_q appears as the current quark mass difference in the basic Lagrangian or Hamiltonian:

$$H = H_{QED} + H_{QCD} \cdot \qquad (6.1)$$

Here H_{QED} is the usual electromagnetic current-current Hamiltonian due to an internal photon loop which we considered in chapter 4, while

$$H_{QCD}(\Delta I=1) = -(\frac{1}{2})\Delta m_q\ (\bar{u}u-\bar{d}d) = -\Delta m_q\ S_3 \quad . \qquad (6.2)$$

(Here Δm_q is the current quark mass difference by definition and $S_3 = (\frac{1}{2})(\bar{u}u-\bar{d}d) = \bar{q}(\frac{\lambda_3}{2})\ q)$. What about the effect of H_{QCD} on the π-N coupling constants? One way such an effect can be taken into account is by observing that H_{QCD} can give rise to $\eta,\eta'-\pi^0$ mixings. The effect due to such mixings on the meson exchange potential and that of the latter on the scattering lengths has been calculated previously (refs.8 and 9). However, these calculations in effect take into account the modification due to $\eta,\eta'-\pi^0$ mixings on the one meson exchange potential only. On the other hand if one considers the effect of H_{QED} on the pion-nucleon coupling constants, one observes the importance of this effect on TPEP, namely an almost complete cancellation between Δa_{cc}^{TPEP} and Δa_{cc}^{OPEP} for CIB and $\delta a_{cc}^{TPEP} \simeq 3\delta a_{cc}^{TPEP}$ for CSB. In view of this, it is important that the effect of H_{QCD} on $\eta - \pi^0$ and $\eta' - \pi^0$ mixings should also be studied in TPEP. Thus it should be more consistent to add the effect of (6.2) on the pion-nucleon coupling constants to that of H_{QED} (which has previously been calculated) and then study the combined effect

of (6.1) on δa_{cc}^{TPEP} and δa_{cc}^{OPEP} for CSB. [It is for this reason that we do not include the contribution of H_{QED} in $\eta,\eta'-\pi^0$ mixings for the calculation of their effect on the pion-nucleon coupling constants] . We indeed also find for H_{QCD} that $\delta_{cc}^{TPEP} \approx 3\delta a_{cc}^{OPEP}$. Our results are summarized in table 6.1 and in sec.3 we discuss the implications of these results and summarize our conclusions.

6.2 - CSB POTENTIAL WITH QCD CORRECTIONS TO PION-NUCLEON COUPLING CONSTANTS

We write the $\pi^0 pp$ and $\pi^0 nn$ coupling constants as

$$g_{\pi^0 pp} = [\ 1 - (\tfrac{1}{2})(\delta_{QCD})\ ]\ g_\pi$$

$$g_{\pi^0 nn} = [\ 1 + (\tfrac{1}{2})(\delta_{QCD})\ ]\ g_\pi\ . \qquad (6.3)$$

Note that the definition of Eqs.(6.3) are similar to Eqs.(4.4). In the above equation, g_π and f are given by $\frac{g_\pi^2}{4\pi} = 14.3$ and $f^2 = (\frac{m_\pi}{2m_N})^2 \frac{g_\pi^2}{4\pi} = 0.08$. δ_{QCD} is the correction due to H_{QCD} and is discussed below. As remarked earlier H_{QCD} leads to $\eta, \eta' - \pi^0$ mixings as shown in Fig.7, which in turn give rise to corrections to the pion-nucleon coupling constants. Before considering the QCD correction to the coupling constants, let us first consider the total Hamiltonian in terms of the quark masses, i.e.

$$H = m_u\ \bar{u}u + m_d\ \bar{d}d + m_s\ \bar{s}s, \qquad (6.4)$$

where m_u, m_d and m_s are the masses of the up, down and strange quark respectively.

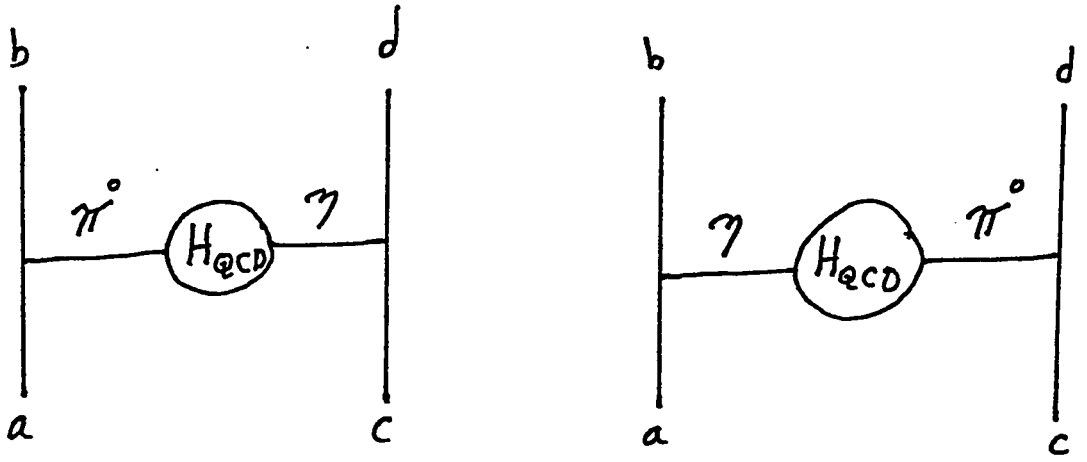

FIGURE 7

Feynman diagram for the η–π mixing that results from H_{QCD}.

Rearranging some of the terms of Eq.(6.4), we get:

$$H = \frac{(m_u+m_d)}{2}(\bar{u}u+\bar{d}d) + \frac{(m_u-m_d)}{2}(\bar{u}u+\bar{d}d) + m_s\,\bar{s}s, \qquad (6.5)$$

or

$$H = (\frac{1}{3})(m_u+m_d+m_s)(\bar{u}u+\bar{d}d+\bar{s}s) + (\frac{1}{3})(\bar{m}-m_s)(\bar{u}u+\bar{d}d-2\bar{s}s)$$

$$+(\frac{1}{2})(m_u-m_d)(\bar{u}u-\bar{d}d)$$

which leads to

$$H = \bar{\bar{m}}(\bar{u}u+\bar{d}d+\bar{s}s) + (\frac{2}{\sqrt{3}})(\bar{m}-m_s)(\frac{\sqrt{3}}{2})(\bar{u}u+\bar{d}d-2\bar{s}s)$$

$$+(\frac{1}{2})(m_u-m_d)(\bar{u}u-\bar{d}d) \qquad (6.6)$$

where $\bar{\bar{m}} = \frac{(m_u+m_d+m_s)}{3}$ and $\bar{m} = \frac{(m_u+m_d)}{2} = m_{ns}$.

Now put

$$S_3 = \bar{q}(\frac{\lambda_3}{2})\,q\,,$$

and (6.7)

$$S_8 = \bar{q}(\frac{\lambda_8}{2})\,q\,,$$

where $\bar{q} = (\bar{u}\ \ \bar{d}\ \ \bar{s})$, $q = \begin{bmatrix} u \\ d \\ s \end{bmatrix}$ (6.8)

and λ_3 , λ_8 are two of generators of SU(3) group. Thus finally we can write (6.6) as

$$H = \tilde{\bar{m}}(\bar{u}u+\bar{d}d+\bar{s}s)+(\frac{2}{\sqrt{3}})(\bar{m}-m_s)\ S_8 + (m_u-m_d)\ S_3 \quad . \qquad (6.9)$$

The first term of Eq.(6.9) is SU(3) symmetry invariant, and (m_u-m_d) is the SU(2) isospin breaking term, while $(m_{ns}-m_s)$ is the SU(3) breaking term. If we assume that $m_u = m_d = m_s$, then SU(3) will not be broken, and if we assume $m_u = m_d$, then SU(2) is not broken.

Now in the quark model, the Gell-Mann-Okubo mass splittings are given by

$$H_{ns} = (\frac{2}{\sqrt{3}})(\bar{m}-m_s)\ S_8 \qquad (6.10)$$

Our $H_{QCD}^{\Delta I=1}$ is $(m_u-m_d)\ S_3$. Since S_3 and S_8 belong to the same octet, we have

$$\langle\pi^0|S_3|\eta_1\rangle = d_{330}\ s_d = (\frac{\sqrt{2}}{3})\ s_d \quad ,$$

$$\langle\pi^0|S_3|\eta_8\rangle = d_{338}\ s_d = (\frac{1}{\sqrt{3}})\ s_d \quad . \qquad (6.11)$$

The reduced matrix element s_d is now determined by the Gell-Mann-Okubo mass splitting:

$$\Delta m_k^2 = \left(\frac{2}{\sqrt{3}}\right)(\bar{m}-m_s)\ d_{\frac{4+i5}{\sqrt{2}}\cdot\frac{4-i5}{\sqrt{2}}\cdot 8}\ s_d\ , \tag{6.12}$$

$$\Delta m_\pi^2 = \left(\frac{2}{\sqrt{3}}\right)(\bar{m}-m_s)\ d_{338}\ s_d \tag{6.13}$$

and from the equations we get

$$s_d = -\frac{(m_k^2 - m_\pi^2)}{(m_{ns} - m_s)}\ . \tag{6.15}$$

From the Feynman diagram shown in Fig.7, using the last term of the Hamiltonian eqn.(6.9), we find the corrections to the pion-nucleon coupling constants as follows:

$$\left(\frac{\delta g}{g}\right)_\pi = \frac{(\delta g)_{\pi^0 pp}}{g_\pi}$$

$$= -\left\{ \frac{g_{\eta\pi}}{(m_\eta^2 - m_\pi^2)} + \frac{g_{\eta'\pi}}{(m_{\eta'}^2 - m_\pi^2)} \times \frac{g_{\eta' pp}}{g_{\eta pp}} \right\} \frac{g_{\eta pp}}{g_\pi} \tag{6.16a}$$

$$= -\frac{(g)_{\pi^0 nn}}{g_\pi}\ , \tag{6.16b}$$

where

$$g_{\eta\pi} = -(m_d - m_u)\ \langle \pi^0 | S_3 | \eta \rangle$$

$$g_{\eta'\pi} = -(m_d - m_u)\ \langle \pi^0 | S_3 | \eta' \rangle\ . \tag{6.17}$$

The physical states η and η' are linear combinations of η_8 (member of SU(3) octet) and η_1 (SU(3) singlet) and are given by Eqs(2.57) and (2.58). Using Eqs(2.57),(2.58),(6.11) and (6.17) we get

$$g_{\eta\pi} = (m_u - m_d) \{ \cos\theta\, d_{338} - \sin\theta\, d_{330} \} s_d$$

$$= (m_u - m_d) \{ \cos\theta - \sqrt{2} \sin\theta \} \frac{1}{\sqrt{3}} s_d \quad . \qquad (6.18)$$

Similarly for $g_{\eta'\pi}$ we find

$$g_{\eta'\pi} = (m_u - m_d) \{ \sin\theta + \sqrt{2} \cos\theta \} \frac{1}{\sqrt{3}} s_d \quad . \qquad (6.19)$$

Using Eqs (6.18) and (6.19), we get from (6.16)

$$(\frac{\delta g}{g})_\pi = \varepsilon \, [1+ \{\frac{(m_\eta^2 - m_\pi^2)}{(m_{\eta'}^2 - m_\pi^2)}\} \tan(\phi) \times$$

$$\times (\frac{g_{\eta' pp}}{g_{\eta pp}}) \,] \times 3 \cos(\theta) \, (\frac{g_{\eta pp}}{g_\pi}) \, , \qquad (6.20)$$

ϕ is the mixing angle between $|\eta_s> = |\bar{s}s>$ and $|\eta_{ns}> = (\frac{1}{2})|\bar{u}u+\bar{d}d>$:

$$\cos(\phi) = (\frac{1}{\sqrt{3}}) \, [\cos(\theta) - \sqrt{2} \sin(\theta) \,] \qquad (6.21)$$

$$\sin(\phi) = (\frac{1}{\sqrt{3}}) \; [\; \sin(\theta) + \sqrt{2}\cos(\theta) \;]$$

and

$$\varepsilon = (\frac{\sqrt{3}}{4})[\frac{m_d - m_u}{m_s - m}] \; \{ \; 1 + \frac{\Delta}{3} \; \} \qquad (6.22a)$$

$$\Delta = \frac{(4m_k^2 - m_\pi^2 - 3m_\eta^2)}{(m_\eta^2 - m_\pi^2)} \; . \qquad (6.22b)$$

The parameter ε is evaluated by using the numerical values (ref.11)

$[\frac{(m_d - m_u)}{(m_s - \bar{m})}] = (\frac{1}{43.5})$ and $\Delta = 0.21$ giving $\varepsilon \simeq 1.07\times10^{-2}$. ε essentially determines the QCD correction to the pion-nucleon coupling constants. The quark-line rule, namely that the $|\eta_s> = |\bar{s}s>$ component of $|\eta>$ and $|\eta'>$ is not coupled to nucleons and by using Eqs(2.57) and (2.58), gives (ref.9)

$$\frac{g_{\eta'}}{g_\eta} = \frac{g_{\eta' pp}}{g_{\eta pp}} = \tan(\phi) \qquad (6.23)$$

Using Eqs(6.3), (6.20-22) and (6.23), we have finally

$$\delta_{QCD} = -2\;\varepsilon[1+\frac{(m_\eta^2 - m_\pi^2)}{(m_{\eta'}^2 - m_\pi^2)}\tan^2(\phi)]\times 3\cos(\phi)(\frac{g_\eta}{g_\pi}) \; . \qquad (6.24)$$

Using the convention of ref.12 for TPEP, we find CSB

corrections to OPEP and TPEP due to δ_0 and δ_{QCD} { as derived in appendix B, with δ_0 replaced by $\delta_0 + \delta_{QCD}$ } as follows:

$$\Delta V_{CSB} = \Delta V_{CSB}^{OPEP} + \Delta V_{CSB}^{TPEP}$$

$$= [f^2 \frac{\exp(-m_\pi r)}{r} - 2 \{ A(00) + B(00) \}] \, 2 \, \delta \qquad (6.25)$$

where

$$\delta = \delta_0 + \delta_{QCD} \quad . \qquad (6.26)$$

Note that in ref.(3) [A(11) + B(11)] has been used instead of [A(00)+B(00)] but the difference between the two expressions which is due to the $\pi^\pm - \pi^0$ mass difference effect will be of higher order in Eq(6.25).

As we see from Eq.(6.24), δ_{QCD} depends on the mixing angle ϕ or θ and the ratio $\frac{g_\eta}{g_\pi}$. Using again that the $|\eta_s>$ coupling to nucleons is zero and SU(3) symmetry, one obtains (see 2.3)

$$\frac{g_\eta}{g_\pi} = \cos(\phi) \, [3F-D] \qquad (6.27)$$

with F+D = 1, where as discussed in references 8 and 9, one may have $1.7 \leq \frac{D}{F} \leq 3$. Recently values of $-\theta\{\phi\}$ larger {smaller} than the standard value (ref.13) $-\theta \simeq 13^0$ $[\phi \simeq 42^0]$

have been obtained (ref.14) from two photon decays of η and η'. For the numerical results in Table 6.1 we thus employ $13° \leq -\theta \leq 25°$ and $1.7 \leq \frac{D}{F} \leq 3$. In obtaining these results, we have used as in ref.3 the Reid soft core potential (ref.15) to generate the wave function u(r) in the integral of Eq.(4.1).

Before we discuss the implications of these results in the next section, we give the CSB corrections for the η and η' exchange potentials due to H_{QCD}. Let us define

$$g_{\eta pp} = g_\eta (1 + \frac{1}{2} \delta^\eta_{QCD})$$

$$g_{\eta' pp} = g_{\eta'} (1 + \frac{1}{2} \delta^{\eta'}_{QCD}) \qquad (6.28)$$

$$g_{\eta nn} = g_\eta (1 - \frac{1}{2} \delta^\eta_{QCD})$$

$$g_{\eta' nn} = g_{\eta'} (1 - \frac{1}{2} \delta^{\eta'}_{QCD}) \qquad (6.29)$$

where δ_{QCD} can be obtained from the first and second part of Eq.(6.24) by interchanging η, η' by π so that

$$\delta^\eta_{QCD} = -(2\sqrt{3}\varepsilon) \cos(\phi) \frac{g_\pi}{g_\eta} = \delta^{(1)}_{QCD} \left(\frac{g^2_\pi}{g^2_\eta}\right) \qquad (6.30)$$

$$\delta_{QCD}^{\eta'} = \delta_{QCD}^{(2)} \left(\frac{g_\pi^2}{g_\eta^2}\right)$$

where

$$\delta_{QCD}^{(1)} = -\varepsilon\ 2\sqrt{3}\ \cos(\phi)\ \left(\frac{g_\eta}{g_\pi}\right) \tag{6.31a}$$

$$\delta_{QCD}^{(2)} = \left[\frac{(m_\eta^2 - m_\pi^2)}{m_{\eta'}^2 - m_\pi^2)}\right]\tan^2(\phi)\ \delta_{QCD}^{(1)} \tag{6.31b}$$

so that

$$\delta_{QCD} = \delta_{QCD}^{(1)} + \delta_{QCD}^{(2)} \quad . \tag{6.32}$$

Using Eqs.(6.24), and (6.29-33) we obtain for CSB in OPEP and One η or η' Exchange Potential (Oη,η'EP) the following expression:

$$\Delta V_{CSB}^{OBEP}(QCD) = f^2 \left\{ \left(\frac{\exp(-m_\pi r)}{r}\right) 2\delta_{QCD} - \left(\frac{m_\eta}{m_\pi}\right)^2 \left(\frac{\exp(-m_\eta r)}{2}\right) 2\delta_{QCD}^{(1)} \right.$$

$$\left. -\left(\frac{m_{\eta'}}{m_\pi}\right)^2 \left[\frac{\exp(-m_{\eta'} r)}{r}\right] 2\ \delta_{QCD}^{(2)} \right\} \quad . \tag{6.33}$$

In Fig.9, we display the integrand of integral (4.1) for OPEP and OBEP, the integrals of which give the corresponding

contributions to δa in Table 6.1. As is seen from Table 6.1 and as was also found in refs. 8 and 9, there is a cancellation between CSB contributions in OPEP and $O\eta\eta'EP$. This is brought about by the relation (6.31) almost independent of values of $-\theta$ and $\frac{g_\eta}{g_\pi}$, although the cancellation tends to be more complete for $-\theta \simeq 13°$ for a given value of D/F.

6.3 - DISCUSSION AND CONCLUSIONS:

From Table 6.1, where we also give the QED-CSB contribution (ref.3) δ_0 to δa for both OPEP and TPEP for comparison, one observes the following:

(i) $$(\delta a)^{TPEP}_{QED} \simeq 3\ (\delta a)^{OPEP}_{QED}\ , \qquad (6.34)$$

with

$$(\delta a)_{QED} = 1.2\ fm = \delta a^{OPEP} + \delta a^{TPEP}$$

(ii) $$(\delta a)^{TPEP}_{QCD} \simeq 3\ (\delta a)^{OPEP}_{QCD}\ , \qquad (6.35)$$

the sign for both of these expressions is opposite to the corresponding QED expressions.

(iii) There is strong cancellation between $(\delta a)^{OPEP}_{QCD}$ and $(\delta a)^{O\eta,\eta'EP}_{QCD}$ essentially because of the relation (6.31) which shows that

$$\delta_{QCD}(\pi) = \delta^{(1)}_{QCD}(\eta) + \delta^{(2)}_{QCD}(\eta')$$

(iv) Unless there is strong cancellation between $(\delta a)^{TPEP}$ and the corresponding CSB contributions from two boson $[\pi^0 , (\eta,\eta')]$ and $[(\eta,\eta'), (\eta,\eta')]$ potential, $(\delta a)^{TPEP}_{QCD}$ is too large to be consistent with the small experimental value for $(\delta a)_{CSB}$ except when $D/F \simeq 2.8$ implying a very small value for $(\frac{g_\pi^2}{4\pi}) \simeq 0.02$.

We now discuss the last point in some detail. Using again the convention of ref.12 so that denoting the (η,π^0) and (η,η') etc. exchange contributions from uncrossed and crossed diagrams as shown in Fig.8 can be written as follows:

$$\bar{A}(0,\eta) = (\frac{g_\eta^2}{g_\pi^2})\ A(0,\eta)$$

$$\bar{A}(\eta,\eta) = (\frac{g_\eta^2}{g_\pi^2})\ A(\eta,\eta) \ , \qquad (6.36)$$

and similarly for B and η'. Thus we can write

$$\delta V^{TBEP}_{CSB} = -2\ [\ A(0,0) + B(0,0)\ ]\ 2\ \delta_{QCD}$$

$$+2\ [\ A(O,\eta) + B(O,\eta)\ ]\ [\ -\ (\frac{g_\eta^2}{g_\pi^2})\delta_{QCD} + \delta_{QCD}^{(1)}\ ]$$

$$+2\ [\ A(\eta,\eta) + B(\eta,\eta)\ ]\ [\ 2\ (\frac{g_\eta^2}{g_\pi^2})\delta_{QCD}^{(1)} + \ ]$$

$$+\quad \eta \rightarrow \eta' \text{ and } \qquad \delta_{QCD}^{(1)} \rightarrow \delta_{QCD}^{(2)} \quad . \qquad (6.37)$$

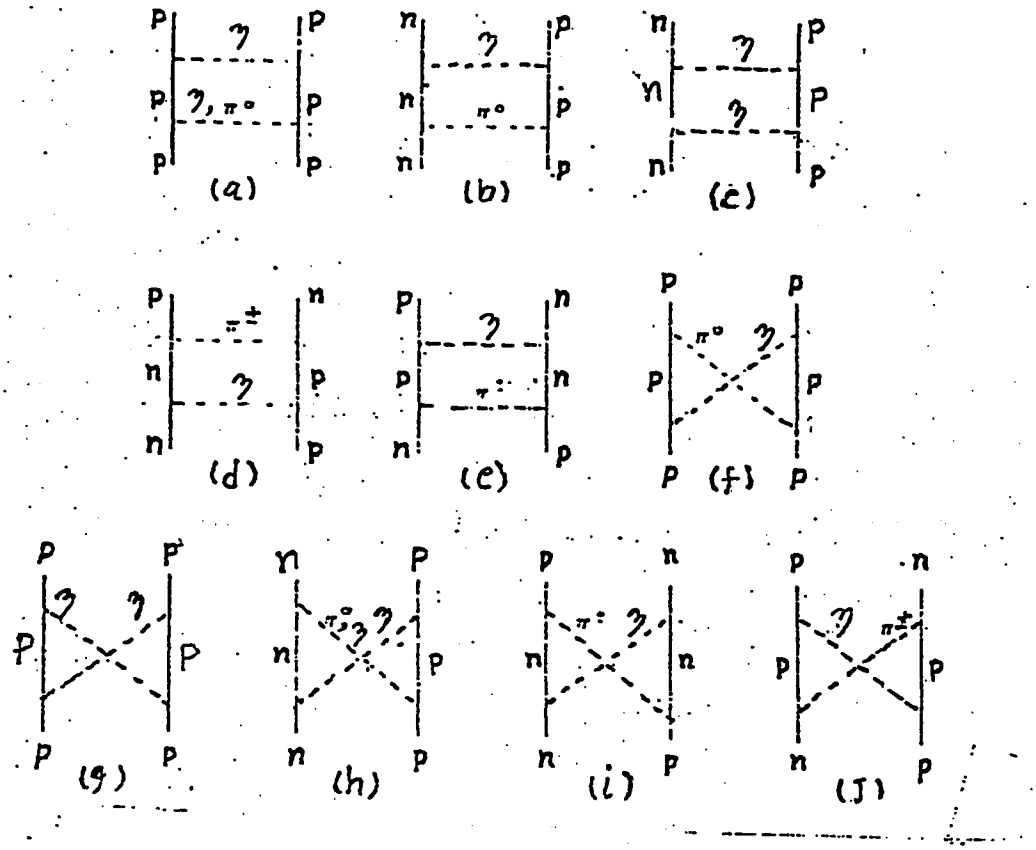

FIGURE 8

Feynman diagrams of two boson exchange $\pi^0-\eta$. Graphs (a) to (e) reprsent uncrossed TBEP diagrams, and graphs (f) to (j) reprsent the crossed TBEP diagrams Note that we can write other possible diagrams of TBEP by considering $\eta\rightarrow\eta'$ and $\pi^0\rightarrow\eta'$.

For OBEP in Eq.(6.33), assuming that π,η and η' dependent terms contribute equally to δa the cancellation is due to (6.32). This assumption for OBEP is supported by the results in Table 6.1. Making, therefore, a similar assumption that A and B do not depend very much on the π,η and η' masses, we obtain from Eq.(6.37), on using Eq.(6.32),

$$\Delta V_{CSB}^{TBEP} = -4\ QCD + [\{-2(\frac{g_\eta^2}{g_\pi^2}) - 2(\frac{g_{\eta'}^2}{g_\pi^2})\}\delta_{QCD} + 2\delta_{QCD}^{(1)} + 2\delta_{QCD}^{(2)}\ ]$$

$$+2\ [\ (\frac{g_\eta^2}{g_\pi^2}) + (\frac{g_{\eta'}^2}{g_\pi^2})\ ][\delta_{QCD}^{(1)} + \delta_{QCD}^{(2)}\ ]$$

$$+2\ (\frac{g_\eta^2}{g_\pi^2})\delta_{QCD}^{(1)} + (\frac{g_{\eta'}^2}{g_\pi^2})\delta_{QCD}^{(2)}$$

$$= -2\ \{1-(\frac{g_\eta^2}{g_\pi^2})\}\delta_{QCD}^{(1)} - 2\{1-(\frac{g_{\eta'}^2}{g_\pi^2})\}\delta_{QCD}^{(2)} \qquad (6.38)$$

Thus we see that for values of $(\frac{g_\eta}{g_\pi})$ and $(\frac{g_{\eta'}}{g_\pi})$ in Table 6.1 we do not see a severe cancellation for TBEP similar to the one for OBEP. If this conjecture is true, then one can conclude that the small experimental value for $(\delta a)_{CSB}$ strongly constrains the η-nucleon coupling constant to be very small ($\simeq 0.02$). In any case the QCD-CSB contribution to TPEP is

important and study of CSB effect due to H_{QCD} through $\pi^0-\eta$, η' mixings appears to strongly constrain the η-nucleon coupling constant to be very small.

The QCD correction to coupling constants contributing to CI is given by conidering Fig.3(a→c) and using Eq.(6.3):

$$V_{np} = -(1+\frac{\delta_{QCD}}{2})(1-\frac{\delta_{QCD}}{2})\ C(0) + 2\ C(1) \qquad (6.39a)$$

$$V = \frac{1}{2}[(1+\frac{\delta_{QCD}}{2})^2\ C(0) + (1-\frac{\delta_{QCD}}{2})^2\ C(0)] = \frac{1}{2}[V_{nn}+V_{pp}] \qquad (6.39b)$$

Thus

$$\Delta V_{CI} = -2\ C(0) + 2\ C(1) - \frac{\delta^2_{QCD}}{4}\ C(0) \qquad (6.39c)$$

neglecting higher order term $\delta^2_{QCD} << 1$, we have

$$\Delta V^{QCD}_{CI,CC} \simeq 0$$

TABLE 6.1

QED and QCD CSB contributions to the scattering lengths:

$$\delta a = a_{pp} - a_{nn}$$

Type of correction	$\theta°$	$\phi°$	$\frac{D}{F}$	$\frac{g_{pp\eta'}}{g_{pp\eta}}$	$\frac{g_{pp\eta}}{g_{pp\pi°}}$	OPEP δa(fm)	TPEP δa(fm)	$O\eta\eta'EP$ δa(fm)
QED Ref(3)	~	~	~	~	~	0.283	0.927	~
QED	-13	42	1.7	0.900	0.358	-1.621	-5.304	1.659
	-19	36	-	0.727	0.390	-1.748	-5.718	1.890
	-25	30	-	0.577	0.477	-1.862	-6.089	2.101
QCD	-13	42	2.0	0.900	0.248	-1.123	-3.674	1.149
	-19	36	-	0.727	0.270	-1.210	-3.959	1.309
	-25	30	-	0.577	0.289	-1.290	-4.220	1.456
QCD	-13	42	2.8	0.900	0.039	-0.177	-0.578	0.181
	-19	36	-	0.727	0.043	-0.193	-0.630	0.209
	-25	30	-	0.577	0.046	-0.205	-0.672	0.323

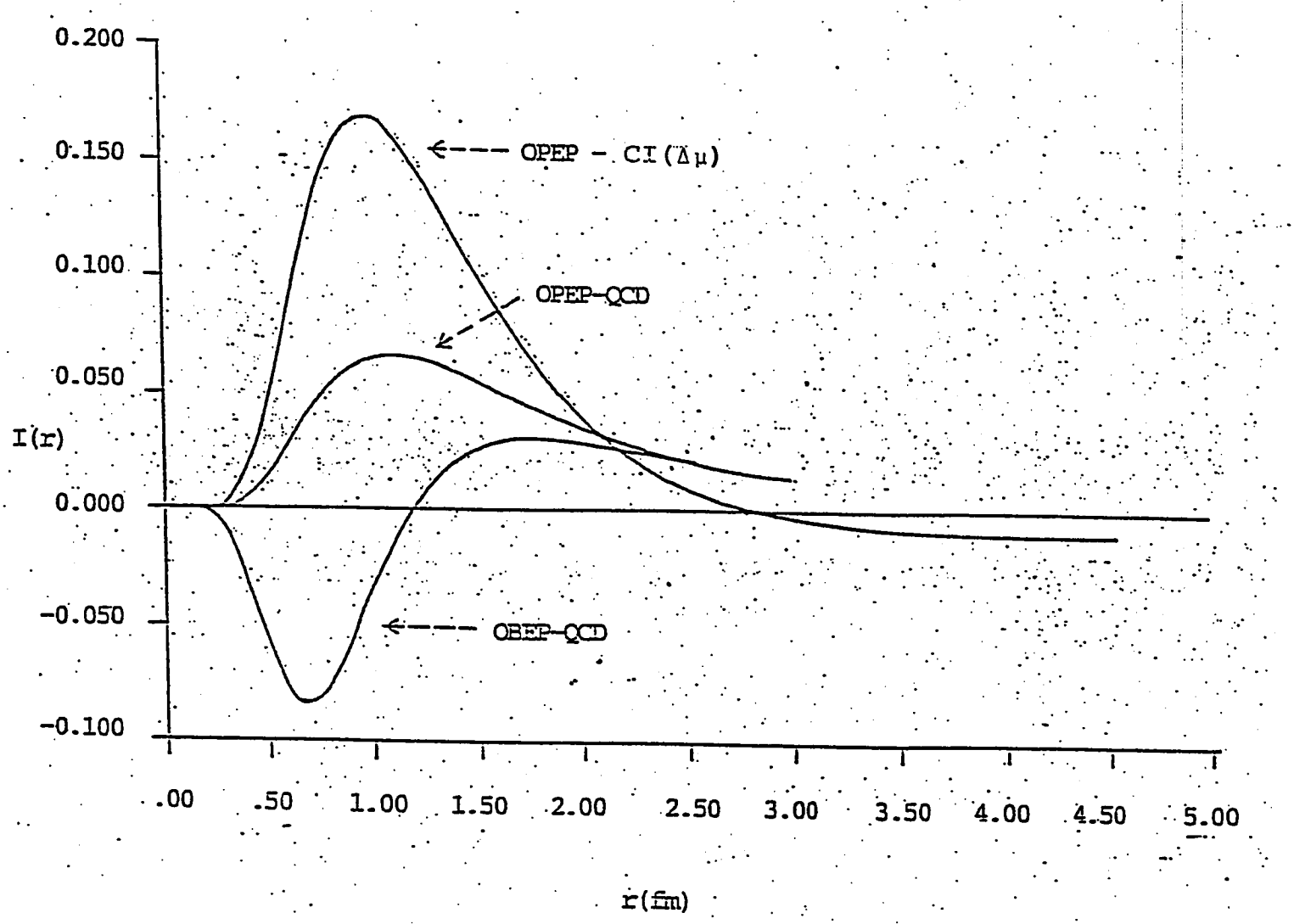

FIGURE 9

Integrands of equ.(4.1) for indicated potential with $\phi = 42^0$ and $\frac{D}{F} = 2$ in QCD. The integrand of OPEP- $\Delta\mu$ is shown for comparision.

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APPENDIX A

In this appendix, we will derive different terms of the TPEP. Referring to Fig.2, we will be using the same notation as in ref(12) where each potential is denoted by $V(PPII_3)$ where PP stands for two pions, I=1,0 for the isospin and I_3=+,0,- for the third component of isospin. The uncrossed diagrams are denoted by A and crossed diagrams by B. The two amplitudes A or B are assigned two numbers 0 or 1 where 0 stands for π^0 and 1 for $\pi^{\pm}$. The relative coupling constants are shown at each vertex of Fig.3, as given in chapter 4.
From Fig.3 (d),

$$V_A(PP1+) = A(00) \tag{1A}$$

$$V_A(PP10) = A(00)+(\sqrt{2})^4 A(11)-(\sqrt{2})^2 A(01)-(\sqrt{2})^2 A(10). \tag{2A}$$

From Fig.3 (i) and (j), we get

$$V_B(PP1+) = B(00) + (\sqrt{2})^4 B(11), \tag{3A}$$

$$V_B(PP10) = B(00) + (\sqrt{2}) B(01) + (\sqrt{2})^2 B(10), \tag{4A}$$

Note that V(PP1-) = V(PP1+) in (1A) and (3A).

Thus we see that the complete V(PP1+) satisfies

$$V(PP1+) = V_A + V_B$$

$$V(PP1+) = A(00) + B(00) + 4B(11) \qquad (5A)$$

and also V(PP10) satisfies

$$V(PP10) = A(00) + 4A(11) - 4A(10) + B(00) + 4B(10), \qquad (6A)$$

Note that A(10) = A(01) and B(10) = B(01) and

$$\Delta V_{\Delta m}^{TPEP} = V(PP1+) - V(PP10)$$

$$= 4 \{ \{B(11) - A(11)\} - \{B(10) - A(10)\} \}, \qquad (7A)$$

To derive eqn.(4.15) for TPEP due to the corrections to the coupling constants and using eqn.(4.15), we can see that;

$V_A(PP1+)=A(00)$ (unchanged)

$$V_A(PP1+) = A(00) + \{\sqrt{2(1+\delta_c)}\}^4 A(11) - 4A(01) \qquad (8A)$$

$$V_B(PP1+) = B(00)+\{\sqrt{2(1+\delta_c)}\}^4 B(11) \qquad (9A)$$

$$V_B(PP10) = B(00) + 4B(10), \qquad (10A)$$

so that

$$\Delta V^{TPEP}=\Delta V_{cc}^{TPEP}+\Delta V_{\Delta\mu}^{TPEP} = V_A(PP1+)+V_B(PP1+)-V_A(PP10)-V_B(PP10), \qquad (11A)$$

$$\begin{aligned} &= A(00) + B(00) + 4(1+2\delta_c + \delta_c^2)\, B(11) \\ &\quad -A(00) - 4(1+2\delta_c + \delta_c^2)\, A(11) + 4A(01) \\ &\quad - B(00) - 4B(010) \end{aligned} \qquad (12A)$$

Neglecting higher order terms since $\delta_c^2 \ll 1$.

$$\begin{aligned} \Delta V_{cc}^{TPEP}+\Delta V_{\Delta m}^{TPEP} \simeq\ & 4\{\ \{B(11) - A(11)\}- \{B(01) - A(01)\} \\ & +8\ \delta_c\ \{\ B(11) - A(11)\ \} \end{aligned} \qquad (13A)$$

from (13A),

$$\Delta V_{cc}^{TPEP} = 8\delta_c \{B(11) - A(11)\} \quad (14A)$$

where A and B are given by

$$A(\alpha) = \{G_\pi^4/(4\pi^4)\}\{X_A(\alpha) + Y_A(\alpha)+W_A(\alpha)+Z_A(\alpha) +Z_B(\alpha), \quad (15A)$$

$$B(\alpha) = \{G_\pi^4/(4\pi^4)\}\{X_B(\alpha) + X_C(\alpha)\}, \quad (16A)$$

J=1,2,3,4 represent central, tensor and spin-spin terms. X,Y,W and Z are integrals whose form are given in eqn.'s[19A,---27A] where

G_π^2 = 14.3 (is the N-N coupling constant)

Then

$$A(\alpha)= \{A(1) + \Omega_{so} A(2) + \Omega_T A(3) + \Omega_{ss} A(4)\}(\alpha)$$

$$B(\alpha)= \{B(1) + \Omega_{so} B(2) + \Omega_T B(3) + \Omega_{ss} A(B)\}(\alpha) \quad (17A)$$

where A(1), B(1) arise from the central potential

A(2), B(2) " " the spin orbit potential

A(3), B(3) " " the tensor potential

A(4), B(4) " " the spin - spin potential

Note that Ω_{so} and Ω_T are equal to zero in our case and α stands for the mass of the pion. In case of A(00) or B(00), the μ^0 -mass is used instead of μ. In A(01) or B(01), the M-mass is used where $M^2 = (\mu_0^2+\mu_c^2)$. In A(11) or B(11), the μ^c -mass is used, For more details see (ref.12).

The final forms of A and B which are used in this thesis after making appropriate change of variables suggested by Appendix B in ref(12), are as follows:

$$X_B(1) = 6\pi^3(\frac{\mu}{x}) \int_0^1 \int_0^1 du\, d\beta\, [\frac{u^5}{(1-u^3)(1-\beta^2)}]\times\exp[(\frac{-2}{\sqrt{(1-u^3)(1-\beta^2)}})\times \{ 1+(\frac{m}{u})^2\{\frac{u^6}{(1-u^3)}\}^{1/2}\} \mu r] , \quad (19A)$$

$$X_C(1)=-12\pi^3\ (1/x)(m/\mu)\mu \int_0^1 \int_0^1 du\, d\beta\, \{u^{11}/\{(1-u^3)^{2.5}\ (1-\beta^2)^{1.5}\}\} \times\{(\mu/m)^2 + \{u^6/(1-u^3)\}\ \}^{-1/2}$$

$$\times\exp\{(-2/\{(1-u^3)(1-\beta^2)\}^{1/2})\{1+(m/\mu)^2\{u^6/(1-u^3)\}\}^{1/2}\mu r\}, \quad (20A)$$

$$Y_A(1)=-12\pi^3(1/x)\mu \int_0^1 \int_0^1 du\, d\beta\, \{u^2/\{(1-u^6)(1-\beta^2)\}\}\times$$

$$\times\exp\{(-2/\{(1-u^6)(1-\beta^2)\}^{1/2})(1+m/\mu)^2\{u^6/(1-u^6)\}\}^{1/2}\ \mu r\}, \quad (21A)$$

$$Z_B(1)=72\pi^3(1/x)\mu \int_0^1\int_0^1\int_0^1 du\ d\beta\ dw\ \{u^2w^2/\{(1-u^6)(1-\beta^2)\}\}\times$$

$$\times\exp\{(-2/\{(1-u^6)(1-\beta^2)\}^{1/2})(1+(m/\mu)^2\{u^6/\{w^6(1-u^6)\}\}\}^{1/2}\ \mu r\}, \quad (22A)$$

$$A = A(1) + \Omega_{so}\ A(2) + \Omega_T\ A(3) + \Omega_{ss}\ A(4), \quad (23A)$$

Including the Ω_{ss} term with $\Omega_{ss} = -3$ we then find that

$$\{B(1)-A(1)\} = \{G\pi^4/(4\pi^4)\}\ \{\{\ 2X_B(1) + X_C(1) - Y_A(1) - 3X_B(4)\ \}$$

$$+ 3\ \{W_A(4) + Z_A(4) - Z_B(1)\ \}\}\ , \quad (24A)$$

Note that $X_A(1) = -X_B(1)$ (see ref(12)).

$$X_B(4)=4\pi^3(1/x)(\mu/m)^2\mu \int_0^1\int_0^1 du\ d\beta\ \{1+(m/\mu)^2\{u^6/(1-u^3)\}\}$$

$$\times\{u^5/\{(1-u^3)(1-\beta^2)\}^2\}$$

$$\times\exp\{(-2/\{(1-u^3)(1-\beta^2)\}^{1/2})(1+(m/\mu)^2\{u^6/(1-u^3)\}\}^{1/2}\mu r\}, \quad (25A)$$

$$W_A(4)=12\pi^3(1/x)(\mu/m)^2\mu\int_0^1\int_0^1\int_0^1 du\ d\beta\ dw\ \{u^5/\{w^4\{(1-u^3)(1-\beta^2)\}^2\}$$

$$\times\{1+(m/\mu)^2\{u^6/(1-u^3)\}\}$$

$$\times\exp\{(-2/\{(1-u^3)(1-\beta^2)\}^{1/2})(1+(m/\mu)^2\{u^6/\{w^6(1-u^3)\}\})^{1/2}\ \mu r\}\ ,$$

(26A)

$$Z_A(4)=-24\pi^3(1/x)(\mu/m)^2\mu\int_0^1\int_0^1\int_0^1 du\ d\beta\ dw\ \{u^8/\{w^4\{(1-u^6)(1-\beta^2)\}^2\}$$

$$\times\{1+(m/\mu)^2\{u^6/(1-u^6)\}\}$$

$$\times\exp\{(-2/\{(1-u^6)(1-\beta^2)\}^{1/2})(1+(m/\mu)^2\{u^6/\{w^6(1-u^6)\}\})^{1/2}\ \mu r\}\ ,$$

(27A)

APPENDIX B

To derive the correction due to H_{QED} in TPEP, we use the same notation as in Appendix A. Using Eqn.(4.4) for writing the potentials in TPEP and Fig.2, and noting that

$$V_{pp}(PP1+) \neq V_{nn}(PP1-)$$

$$V_{pp}(PP10) \neq V_{nn}(PP10) \qquad (1B)$$

Eqn.(1B) holds because of relation (4.4), i.e. the couplings are different in n-n and p-p pairs.

so that

$$V_{pp}(pp1+) = (1-\delta_0/2)^2\, A(00) + (1-\delta_0/2)^2\, B(00) + 4B(11)\ , \qquad (2B)$$

$$V_{pp}(pp10) = (1+\delta_0/2)^2\, A(00) + 4A(11) - 4A(10) + (1-\delta_0/2)^2\, B(00) + 4B(11)\ , \qquad (3B)$$

$$V_{nn}(pp1-) = (1+\delta_0/2)^2\, A(00) + (1+\delta_0/2)^2\, B(00) + 4B(11)\ , \qquad (4B)$$

$$V_{nn}(pp10) = (1+\delta_0/2)^2\ A(00) + 4A(11) - 4A(10)$$

$$+ (1+\delta_0/2)^2\ B(00) + 4B(11)\ , \qquad (5B)$$

such that

$$\Delta V_{CSB}^{TPEP}=\{V_{pp}(pp1+)+V_{PP}(PP10)\}-\{V_{nn}(pp1-)+V_{nn}(pp10)\}, \qquad (6B)$$

Neglecting higher order terms in Eqn.(6B), since $\delta_0^2 << 1$, we get,

$$\Delta V_{CSB}^{TPEP} = -4\delta_0\ \{\ A(00) + B(00)\ \}\ , \qquad (7B)$$

APPENDIX C

This subroutine is for solving second order differntial equation, and was taken from SSP library.

```
C     ..................................................................RKGS1030
C                                                                       RKGS1040
      SUBROUTINE RKGS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)               RKGS1050
C                                                                       RKGS1060
C                                                                       RKGS1070
C     DIMENSION Y(1),DERY(1),AUX(8,1),A(4),B(4),C(4),PRMT(1)            RKGS1080
      DOUBLE PRECISION Y(1),DERY(1),AUX(8,1),A(4),B(4),C(4),PRMT(1),    RKGS1080
      X,H
      DO 1 I=1,NDIM
    1 AUX(8,I)=.06666667*DERY(I)                                        RKGS1090
      X=PRMT(1)                                                         RKGS1100
      XEND=PRMT(2)                                                      RKGS1110
      H=PRMT(3)                                                         RKGS1120
      PRMT(5)=0.                                                        RKGS1130
      CALL FCT(X,Y,DERY)                                                RKGS1140
C                                                                       RKGS1150
C     ERROR TEST                                                        RKGS1160
      IF(H*(XEND-X))38,37,2                                             RKGS1170
C                                                                       RKGS1180
C     PREPARATIONS FOR RUNGE-KUTTA METHOD                               RKGS1190
    2 A(1)=.5                                                           RKGS1200
      A(2)=.2928932                                                     RKGS1210
      A(3)=1.707107                                                     RKGS1220
      A(4)=.1666667                                                     RKGS1230
      B(1)=2.                                                           RKGS1240
      B(2)=1.                                                           RKGS1250
      B(3)=1.                                                           RKGS1260
      B(4)=2.                                                           RKGS1270
      C(1)=.5                                                           RKGS1280
      C(2)=.2928932                                                     RKGS1290
      C(3)=1.707107                                                     RKGS1300
      C(4)=.5                                                           RKGS1310
C                                                                       RKGS1320
C     PREPARATIONS OF FIRST RUNGE-KUTTA STEP                            RKGS1330
      DO 3 I=1,NDIM                                                     RKGS1340
      AUX(1,I)=Y(I)                                                     RKGS1350
      AUX(2,I)=DERY(I)                                                  RKGS1360
      AUX(3,I)=0.                                                       RKGS1370
    3 AUX(6,I)=0.                                                       RKGS1380
      IREC=0                                                            RKGS1390
      H=H+H                                                             RKGS1400
                                                                        RKGS1410
```

```
      IHLF=-1                                                          RKGS1420
      ISTEP=0                                                          RKGS1430
      IEND=0                                                           RKGS1440
C                                                                      RKGS1450
C                                                                      RKGS1460
C     START OF A RUNGE-KUTTA STEP                                      RKGS1470
    4 IF((X+H-XEND)*H)7,6,5                                            RKGS1480
    5 H=XEND-X                                                         RKGS1490
    6 IEND=1                                                           RKGS1500
C                                                                      RKGS1510
C     RECORDING OF INITIAL VALUES OF THIS STEP                         RKGS1520
    7 CALL OUTP(X,Y,DERY,IREC,NDIM,PRMT)                               RKGS1530
      IF(PRMT(5))40,8,40                                               RKGS1540
    8 ITEST=0                                                          RKGS1550
    9 ISTEP=ISTEP+1                                                    RKGS1560
C                                                                      RKGS1570
C                                                                      RKGS1580
C     START OF INNERMOST RUNGE-KUTTA LOOP                              RKGS1590
      J=1                                                              RKGS1600
   10 AJ=A(J)                                                          RKGS1610
      BJ=B(J)                                                          RKGS1620
      CJ=C(J)                                                          RKGS1630
      DO 11 I=1,NDIM                                                   RKGS1640
      R1=H*DERY(I)                                                     RKGS1650
      R2=AJ*(R1-BJ*AUX(6,I))                                           RKGS1660
      Y(I)=Y(I)+R2                                                     RKGS1670
      R2=R2+R2+R2                                                      RKGS1680
   11 AUX(6,I)=AUX(6,I)+R2-CJ*R1                                       RKGS1690
      IF(J-4)12,15,15                                                  RKGS1700
   12 J=J+1                                                            RKGS1710
      IF(J-3)13,14,13                                                  RKGS1720
   13 X=X+.5*H                                                         RKGS1730
   14 CALL FCT(X,Y,DERY)                                               RKGS1740
      GOTO 10                                                          RKGS1750
C     END OF INNERMOST RUNGE-KUTTA LOOP                                RKGS1760
C                                                                      RKGS1770
C                                                                      RKGS1780
C     TEST OF ACCURACY                                                 RKGS1790
   15 IF(ITEST)16,16,20                                                RKGS1800
C                                                                      RKGS1810
C     IN CASE ITEST=0 THERE IS NO POSSIBILITY FOR TESTING OF ACCURACY  RKGS1820
   16 DO 17 I=1,NDIM                                                   RKGS1830
   17 AUX(4,I)=Y(I)                                                    RKGS1840
      ITEST=1                                                          RKGS1850
      ISTEP=ISTEP+ISTEP-2                                              RKGS1860
   18 IHLF=IHLF+1                                                      RKGS1870
      X=X-H                                                            RKGS1880
      H=.5*H                                                           RKGS1890
      DO 19 I=1,NDIM                                                   RKGS1900
      Y(I)=AUX(1,I)                                                    RKGS1910
      DERY(I)=AUX(2,I)                                                 RKGS1920
```

```
   19 AUX(6,I)=AUX(3,I)                                                  RKGS1930
      GOTO 9                                                             RKGS1940
C                                                                        RKGS1950
C     IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE                    RKGS1960
   20 IMOD=ISTEP/2                                                       RKGS1970
      IF(ISTEP-IMOD-IMOD)21,23,21                                        RKGS1980
   21 CALL FCT(X,Y,DERY)                                                 RKGS1990
      DO 22 I=1,NDIM                                                     RKGS2000
      AUX(5,I)=Y(I)                                                      RKGS2010
   22 AUX(7,I)=DERY(I)                                                   RKGS2020
      GOTO 9                                                             RKGS2030
C                                                                        RKGS2040
C     COMPUTATION OF TEST VALUE DELT                                     RKGS2050
   23 DELT=0.                                                            RKGS2060
      DO 24 I=1,NDIM                                                     RKGS2070
   24 DELT=DELT+AUX(8,I)*DABS(AUX(4,I)-Y(I))                             RKGS2080
      IF(DELT-PRMT(4))28,28,25                                           RKGS2090
C                                                                        RKGS2100
C     ERROR IS TOO GREAT                                                 RKGS2110
   25 IF(IHLF-10)26,36,36                                                RKGS2120
   26 DO 27 I=1,NDIM                                                     RKGS2130
   27 AUX(4,I)=AUX(5,I)                                                  RKGS2140
      ISTEP=ISTEP+ISTEP-4                                                RKGS2150
      X=X-H                                                              RKGS2160
      IEND=0                                                             RKGS2170
      GOTO 18                                                            RKGS2180
C                                                                        RKGS2190
C     RESULT VALUES ARE GOOD                                             RKGS2200
   28 CALL FCT(X,Y,DERY)                                                 RKGS2210
      DO 29 I=1,NDIM                                                     RKGS2220
      AUX(1,I)=Y(I)                                                      RKGS2230
      AUX(2,I)=DERY(I)                                                   RKGS2240
      AUX(3,I)=AUX(6,I)                                                  RKGS2250
      Y(I)=AUX(5,I)                                                      RKGS2260
   29 DERY(I)=AUX(7,I)                                                   RKGS2270
      CALL OUTP(X-H,Y,DERY,IHLF,NDIM,PRMT)                               RKGS2280
      IF(PRMT(5))40,30,40                                                RKGS2290
   30 DO 31 I=1,NDIM                                                     RKGS2300
      Y(I)=AUX(1,I)                                                      RKGS2310
   31 DERY(I)=AUX(2,I)                                                   RKGS2320
      IREC=IHLF                                                          RKGS2330
      IF(IEND)32,32,39                                                   RKGS2340
C                                                                        RKGS2350
C     INCREMENT GETS DOUBLED                                             RKGS2360
   32 IHLF=IHLF-1                                                        RKGS2370
      ISTEP=ISTEP/2                                                      RKGS2380
      H=H+H                                                              RKGS2390
      IF(IHLF)4,33,33                                                    RKGS2400
   33 IMOD=ISTEP/2                                                       RKGS2410
      IF(ISTEP-IMOD-IMOD)4,34,4                                          RKGS2420
   34 IF(DELT-.02*PRMT(4))35,35,4                                        RKGS2430
```

```
   35 IHLF=IHLF-1
      ISTEP=ISTEP/2                                                    RKGS2440
      H=H+H                                                            RKGS2450
      GOTO 4                                                           RKGS2460
C                                                                      RKGS2470
C                                                                      RKGS2480
C     RETURNS TO CALLING PROGRAM                                       RKGS2490
   36 IHLF=11                                                          RKGS2500
      CALL FCT(X,Y,DERY)                                               RKGS2510
      GOTO 39                                                          RKGS2520
   37 IHLF=12                                                          RKGS2530
      GOTO 39                                                          RKGS2540
   38 IHLF=13                                                          RKGS2550
   39 CALL OUTP(X,Y,DERY,IHLF,NDIM,PRMT)                               RKGS2560
   40 RETURN                                                           RKGS2570
      END                                                              RKGS2580
C*********************************************************************RKGS2590
      DOUBLE PRECISION PRMT(5),AUX(8,2),DERY(2),Y(2),X,C
      EXTERNAL FCT,OUTP
      C=18.2
      PRMT(1)=0.02
      PRMT(2)=4.6*.7
      PRMT(3)=0.05*0.7
      PRMT(4)=0.0001
c     Y(1)=1.0/(C*0.7)
c     Y(2)=1.0+(10.0/(C*0.7))
      Y(1)=0.1
      Y(2)=0.000
      DERY(1)=0.5
      DERY(2)=0.5
      NDIM=2
      CALL RKGS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
      STOP
      END
      SUBROUTINE FCT(X,Y,DERY)
      DOUBLE PRECISION Y(2),DERY(2),H,C,V,X,C1,C2,C5
      H=10.463
      C1=1650.6
      C2=6484.2
      C=938.26/((197.4**2)*(0.7**2))
     DERY(1)=-C*((H*DEXP(-X)+C1*DEXP(-4.0*X)-C2*DEXP(-7.0*X))/(X))*Y(2)2)
      DERY(2)=Y(1)
      RETURN
      END
      SUBROUTINE OUTP(X,Y,DERY,IREC,NDIM,PRMT)
      DOUBLE PRECISION Y(2),DERY(2),PRMT(5),X,V,C1,C2,H
      WRITE(9,2)X,Y(2)
2     FORMAT (1X,2F15.7)
      RETURN
      END
```

This programme is for multi-dimentional integration and was used with the subroutine QB01A taken from HARWEL SUBROTINE LIBRARY.

```
      IMPLICIT REAL*8(A-H,O-Z)                                          CVC00010
      DIMENSION MAXCO(6),ITYPE(6),ICOUNT(6),R(100),U(100),X1(100)       CVC00020
      COMMON R,U,I,MMAX                                                 CVC00030
      ITYPE(1)=0                                                        CVC00040
      ITYPE(2)=0                                                        CVC00050
C     ITYPE(3)=0                                                        CVC00060
C                                                                       CVC00070
      L=48                                                              CVC00080
      N=95                                                              CVC00090
C                                                                       CVC00100
C  SET THE NO. OF INTEGRATIONS                                          CVC00110
      MMAX=2                                                            CVC00120
C  SET THE ERROR REQUIRED                                               CVC00130
      ERROR=0.0001                                                      CVC00140
2     FORMAT(1X,2F15.7)                                                 CVC00150
      DO 10 I=1,N                                                       CVC00160
10       READ(9,2)X1(I),U(I)                                            CVC00170
      DO 20 I=54,N                                                      CVC00180
      R(I)=X1(I)/.7                                                     CVC00190
       CALL QB01A(ERROR,MAXCO,ITYPE,MMAX,F,ICOUNT)                      CVC00200
20    WRITE(7,2)R(I),F                                                  CVC00210
      STOP                                                              CVC00220
      END                                                               CVC00230
C                                                                       CVC00240
C                                                                       CVC00250
C                                                                       CVC00260
C                                                                       CVC00270
C                                                                       CVC00280
      SUBROUTINE LIMITS(M,A,B,G,X,F,FA)                                 CVC00290
      IMPLICIT REAL*8(A-H,O-Z)                                          CVC00300
      DIMENSION R(100),U(100),FA(6),X(6),A1(6),B1(6)                    CVC00310
      COMMON R,U,I,MMAX                                                 CVC00320
      A1(1)=0.000464                                                    CVC00330
      A1(2)=0.1D -9                                                     CVC00340
      B1(1)=.9999996                                                    CVC00350
      B1(2)=.999999                                                     CVC00360
      A=A1(M)                                                           CVC00370
      B=B1(M)                                                           CVC00380
      G=0.0                                                             CVC00390
      FA(M)=1.0                                                         CVC00400
      IF(M.EQ.MMAX+1) THEN                                              CVC00410
C EVALUATE THE FUNCTION TO BE INTEGRATED                                CVC00420
C     F=X(2)*X(3)**2*EXP(-X(1)*X(2)*X(3)*R(I))                          CVC00430
      CP=(14.4**2)*6.0/(3.14159)*139.6                                  CVC00440
```

```
      C=-150.0                                                          CVC00450
      C2=45.2551                                                        CVC00460
      C3=0.70745                                                        CVC00470
      C4=2.0/C2                                                         CVC00480
      C1=1.0                                                            CVC00490
      C6=-2.0*C2                                                        CVC00500
C XB IS MULT. BY 2.0 , SINCE B(1)-A(1) =XB+XC-XA-YA-ZB                  CVC00510
C BUT XB=-XA.  C3=(NUCLEON MASS/M)**2 ,C4=M/CHARGED PION                CVC00520
C                                                                       CVC00530
      T=DLOG(2.0*((X(1)**5)/((1.0-X(1)**3)*(1.0-X(2)**2))))+            CVC00540
      ((-2.0/(DSQRT(1.0-X(1)**3)*DSQRT(1.0-X(2)**2)))*DSQRT(C1+         CVC00550
      (X(1)**6.0)*C2/(1.0-X(1)**3))*R(I)*C3)                            CVC00560
      IF(T.LE.C) THEN                                                   CVC00570
        XB=0.0                                                          CVC00580
      ELSE                                                              CVC00590
        XB=DEXP(T)                                                      CVC00600
      END IF                                                            CVC00610
C                                                                       CVC00620
C                                                                       CVC00630
C                                                                       CVC00640
      T2=DLOG((X(1)**11)/(((1.0-X(1)**3)**2.5)*(1.0-X(2)**2)**1.5)*     CVC00650
      (1.0/(SQRT(C1+C2*(X(1)**6)/(1.0-X(1)**3)))))+                     CVC00660
      ((-2.0/(DSQRT(1.0-X(1)**3)*DSQRT(1.0-X(2)**2)))*DSQRT(C1+         CVC00670
      (X(1)**6.0)*C2/(1.0-X(1)**3))*R(I)*C3)                            CVC00680
      IF(T2.LE.C) THEN                                                  CVC00690
        XC=0.0                                                          CVC00700
      ELSE                                                              CVC00710
        XC=DEXP(T2)                                                     CVC00720
      END IF                                                            CVC00730
C                                                                       CVC00740
C                                                                       CVC00750
      T4=DLOG((X(1)**2)/((1.0-X(1)**6)*(1.0-X(2)**2)))+                 CVC00760
      ((-2.0/(DSQRT(1.0-X(1)**6)*DSQRT(1.0-X(2)**2)))*DSQRT(C1+         CVC00770
      (X(1)**6.0)*C2/(1.0-X(1)**6))*R(I)*C3)                            CVC00780
      IF(T4.LE.C) THEN                                                  CVC00790
        YA1=0.0                                                         CVC00800
      ELSE                                                              CVC00810
        YA1=DEXP(T4)                                                    CVC00820
      END IF                                                            CVC00830
C                                                                       CVC00840
      T5=DLOG((X(1)**5)*(C1+C2*(X(1)**6)/(1.0-X(1)**3))/(((1.0-X(1)**3) CVC00850
      *(1.0-X(2)**2))**2))+                                             CVC00860
      ((-2.0/(DSQRT(1.0-X(1)**3)*DSQRT(1.0-X(2)**2)))*DSQRT(C1+         CVC00870
      (X(1)**6.0)*C2/(1.0-X(1)**3))*R(I)*C3)                            CVC00880
      IF(T5.LE.C) THEN                                                  CVC00890
        XB4=0.0                                                         CVC00900
      ELSE                                                              CVC00910
        XB4=DEXP(T5)                                                    CVC00920
      END IF                                                            CVC00930
C                                                                       CVC00940
C                                                                       CVC00950
```

```
      F=CP*((XB-XB4*C4)/(R(I)*C3)+C6*XC)                                CVC00960
C     XB+2.0*YA1-XB4*C4)/(R(I)*C3)+C6*XC)                               CVC00970
C                                                                       CVC00980
      END IF                                                            CVC00990
      RETURN                                                            CVC01000
      END                                                               CVC01010
C                                                                       CVC01020
```